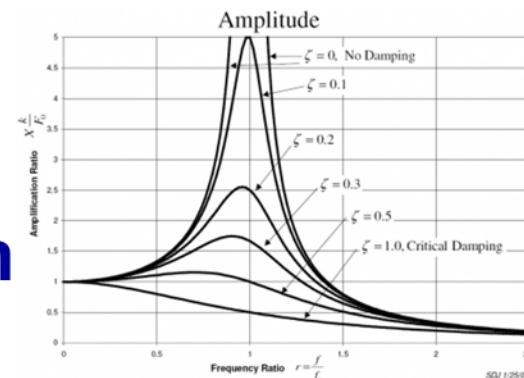
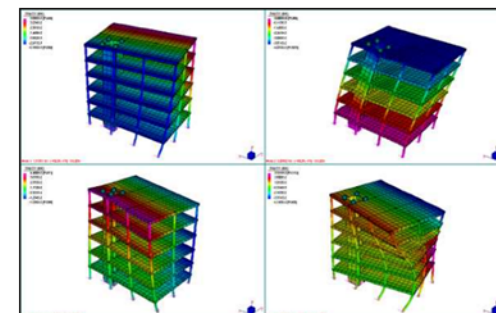


Training seminar for Latvian structural engineers /

Apmācību seminārs Latvijas būvkonstrukciju projektētājiem:

Seismic-resistant structural design / Seismiski izturīgu konstrukciju projektēšana



Module 1: Fundamentals of Seismic Design

Online, 9th April 2025



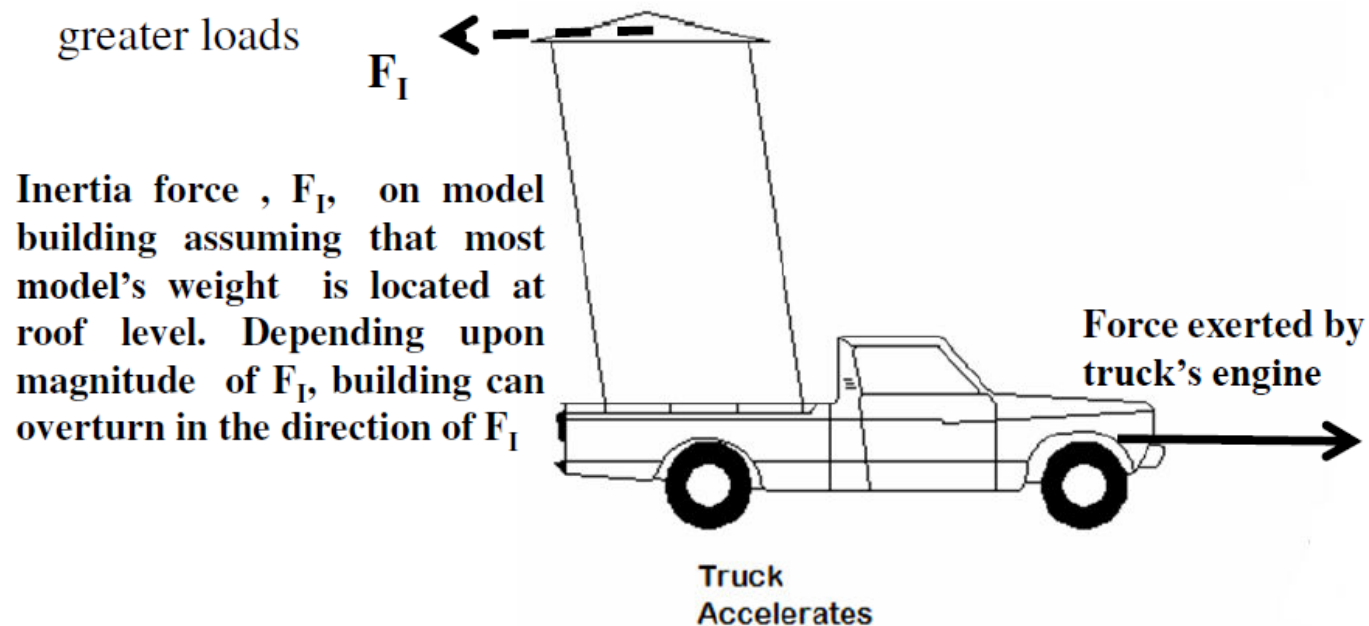
www.lbpa.lv

Massimo Latour

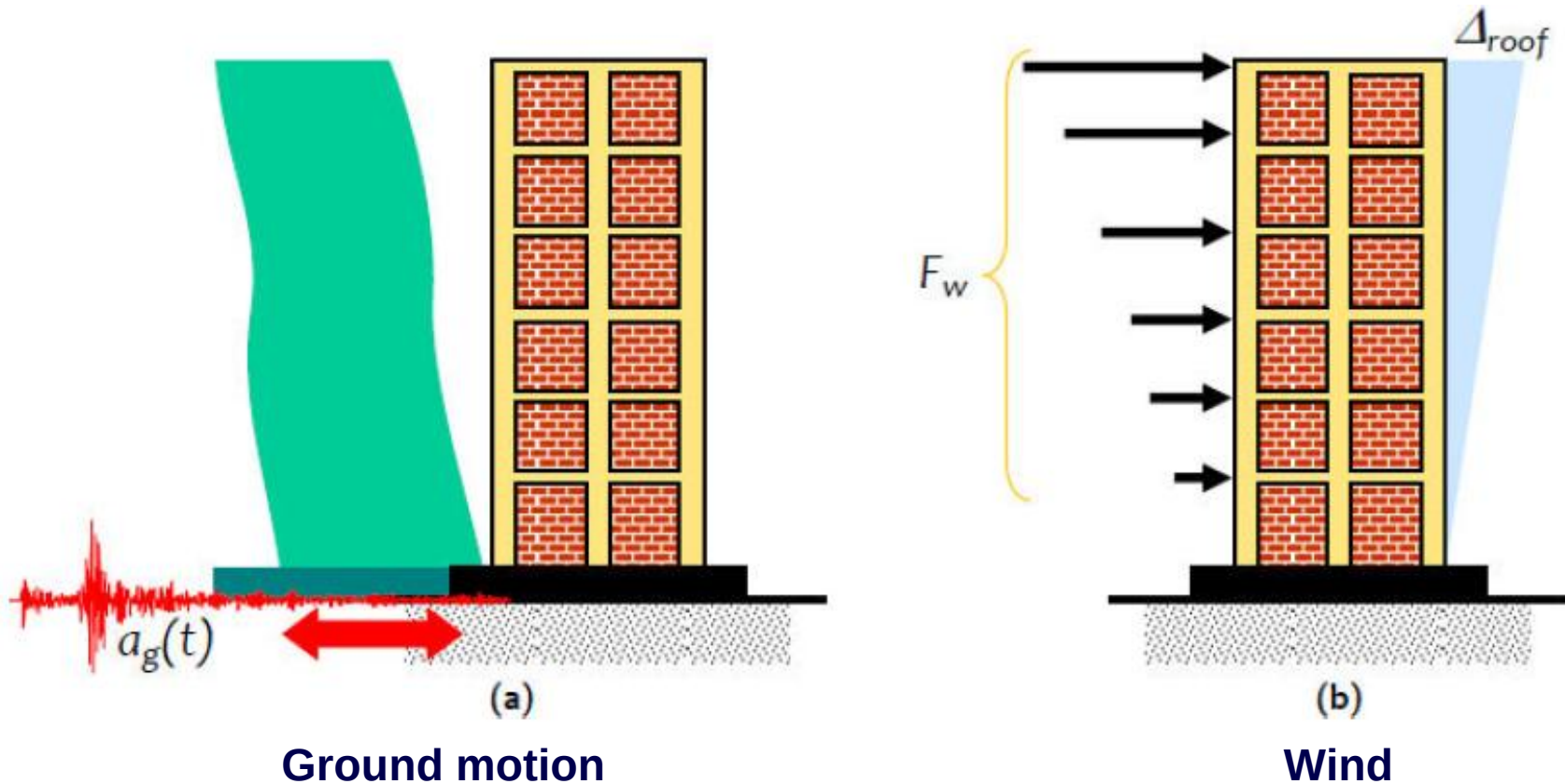
Department of Civil Engineering
University of Salerno - ITALY

Introduction

- Earthquake causes **ground shaking**;
- Ground shaking induces movement of the soil back and forth generating inertia forces in the building. Obviously, the bigger the mass, the bigger the **inertia forces**;



Introduction



Even though the earthquake is a **displacement-type load**, for practical purposes in many cases we try to represent its effects on the structure in terms of **equivalent static forces**.

Introduction



Outside



Inside

OUTLINE OF THE PRESENTATION

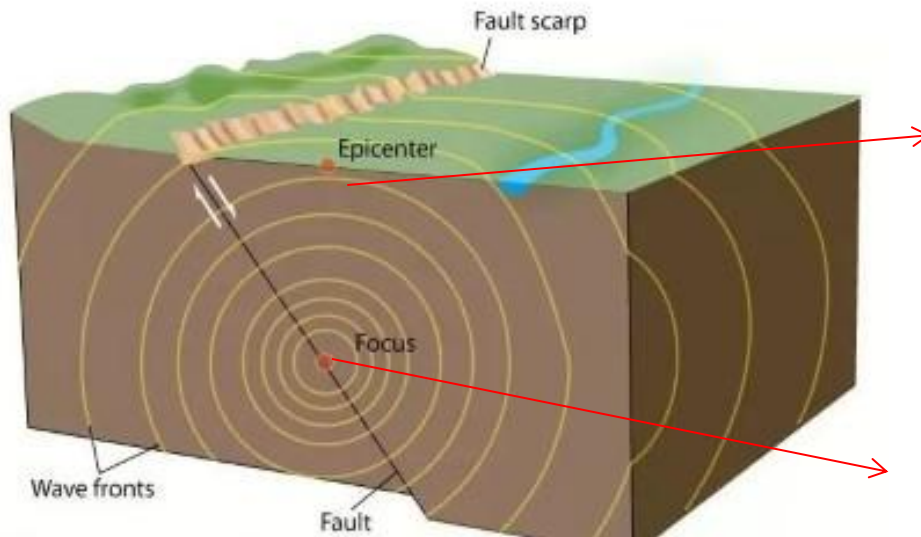
- ❑ **PART1:** Definition of the Hazard
- ❑ **PART2:** Principles of dynamics
- ❑ **PART 3:** The role of ductility

Definition of the Hazard

Definition of the hazard

The earthquake effects and how to measure its intensity

Seismic Waves Radiate from the Focus of an Earthquake



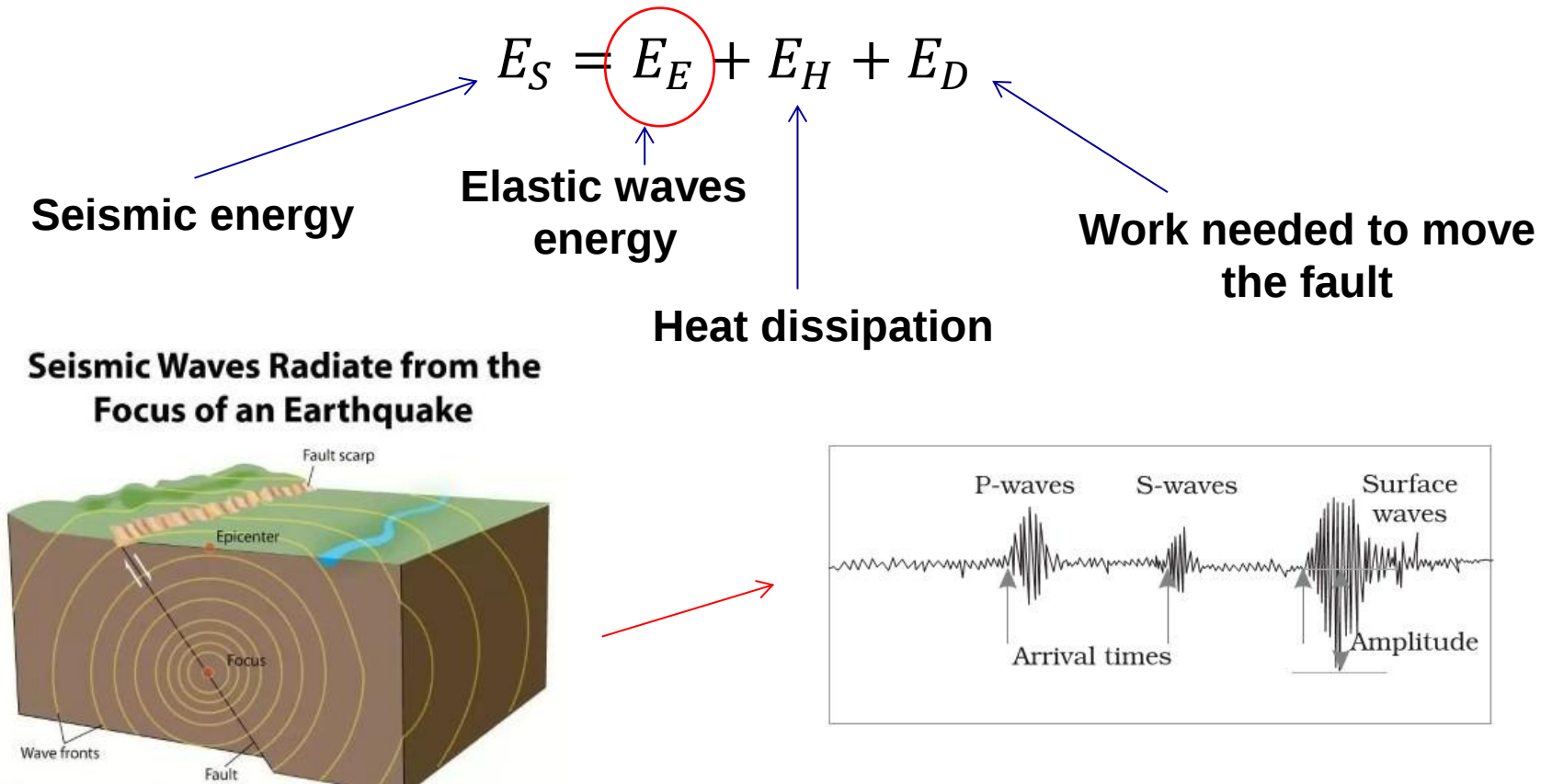
- *Superficial (70 km)*
- *Intermediate (70-300 km)*
- *Deep (>300 km)*

The epicentre is the point on the surface vertically above the focus (maximum damage)

The focus is the place of origin of an earthquake inside the ground

Definition of the hazard

The earthquake effects and how to measure its intensity



Definition of the hazard

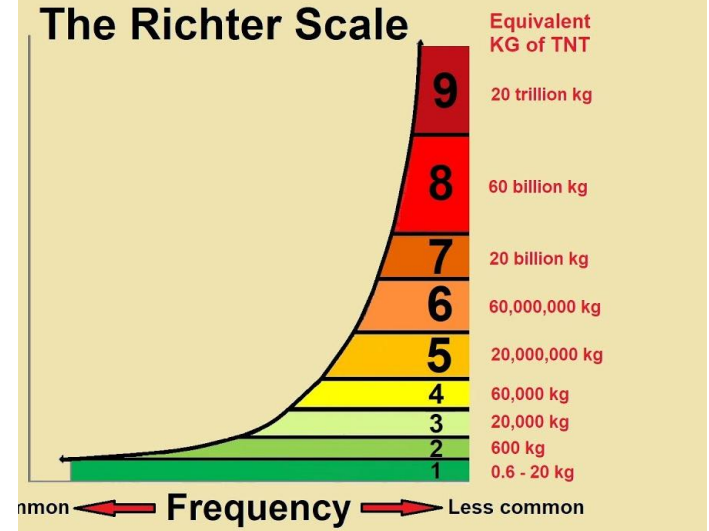
Measures of intensity (Scales)

- Mercalli scale (1902, based on the measure of the effects 1-12) – not objective;
- Richter scale (based on the definition of magnitude, it measures the amplitude recorded at the accelerometer at a certain distance). Magnitude 0 corresponds to a seismic event generating 1 micron of amplitude on the accelerometer at 100 km from the epicentre – correlated to the Energy at the focus ($\log E = 11.8 + 1.5M$);

Modified Mercalli Scale

- I.** Not felt.
- II.** Felt by persons at rest, on upper floors, or favorably placed.
- III.** Felt indoors. Vibration like passing of light trucks.
- IV.** Vibration like passing of heavy trucks.
- V.** Felt outdoors. Small unstable objects displaced or upset.
- VI.** Felt by all. Furniture moved. Weak plaster/masonry cracks.
- VII.** Difficult to stand. Damage to masonry and chimneys.
- VIII.** Partial collapse of masonry. Frame houses moved.
- IX.** Masonry seriously damaged or destroyed.
- X.** Many buildings and bridges destroyed.
- XI.** Rails bent greatly. Pipelines severely damaged.
- XII.** Damage nearly total.

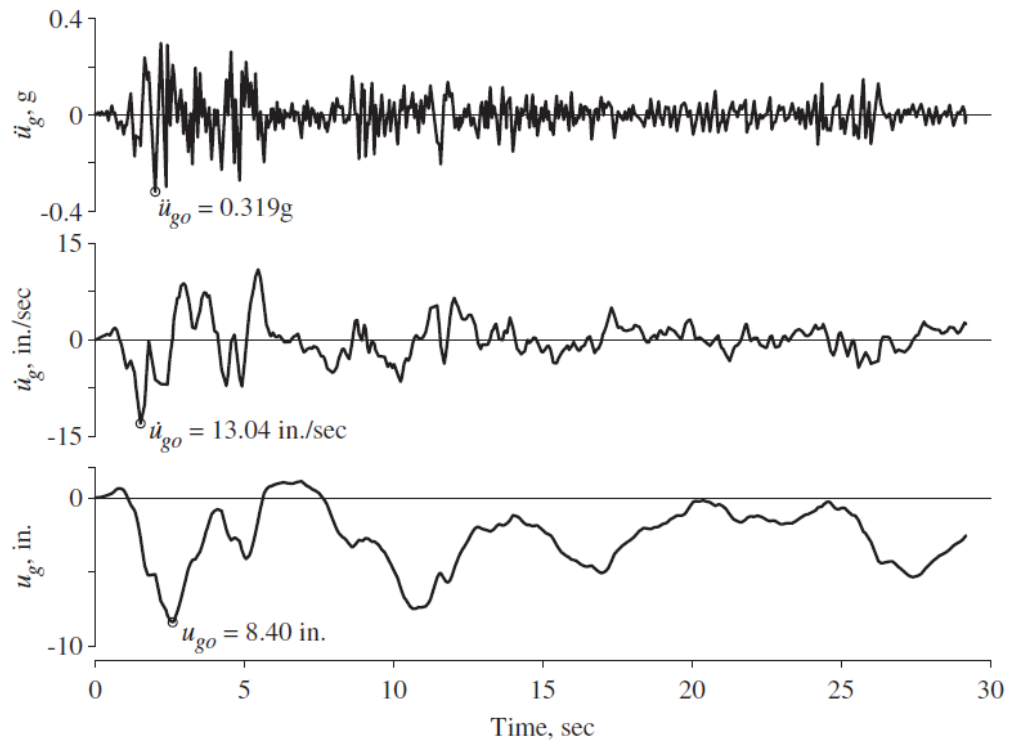
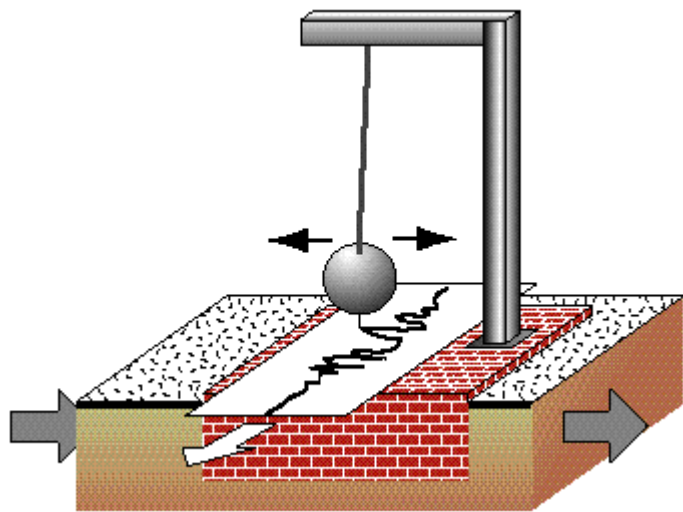
The Richter Scale



Definition of the hazard

Representation of the earthquake at ground level, intensity measures

- **PGA, PGV, PGD:** Peak Ground Acceleration, velocity and displacement;



For structural engineers the full accelerogram is needed

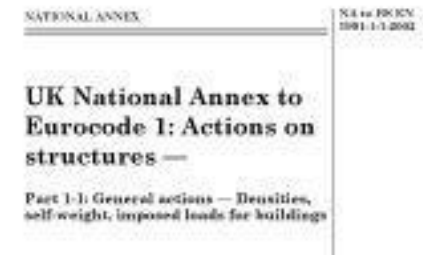
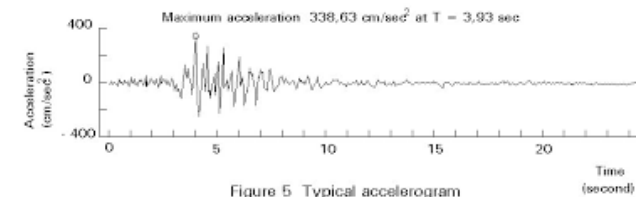
Figure 6.1.4 North-south component of horizontal ground acceleration recorded at the Imperial Valley Irrigation District substation, El Centro, California, during the Imperial Valley earthquake of May 18, 1940. The ground velocity and ground displacement were computed by integrating the ground acceleration.

Definition of the hazard

Seismic Hazard in EN 1998 (Eurocode 8)

Seismic hazard is defined using a **single parameter**:

- **Reference Peak Ground Acceleration (a_{gR}) on Ground Type A** (rock or rock-like soil).
- Each country provides this info in its **National Annex**.
- National authorities divide the country into **seismic zones**, where the hazard (a_{gR}) is considered **uniform** within each zone.
- For each zone, a_{gR} is linked to a **reference probability** of exceedance (Typically for a **10% probability in 50 years** → used for **no-collapse**)



Definition of the hazard

Hazard

As previously mentioned, the **random nature of earthquakes** and the **numerous uncertainties** involved in assessing seismic hazard at a given site make a **probabilistic approach** highly appropriate.

In such analyses, the **seismic events** are typically modeled using a **stationary Poisson process**, which forms the basis of the underlying probabilistic framework.

Probability of exceedance P_R	Time span T_L	Mean return period T_R
20%	10 years	45 years
10%	10 years	95 years
20%	50 years	224 years
10%	50 years	475 years
5%	50 years	975 years
10%	100 years	949 years
5%	100 years	1950 years

Level 1 – Fully Operational

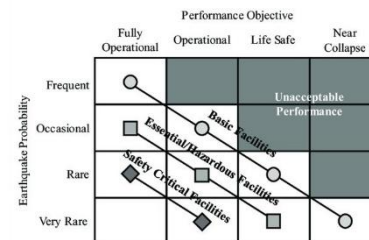
Level 2 – Operational

Level 3 – Life Safe

Level 4 – Near Collapse

Definition of the hazard

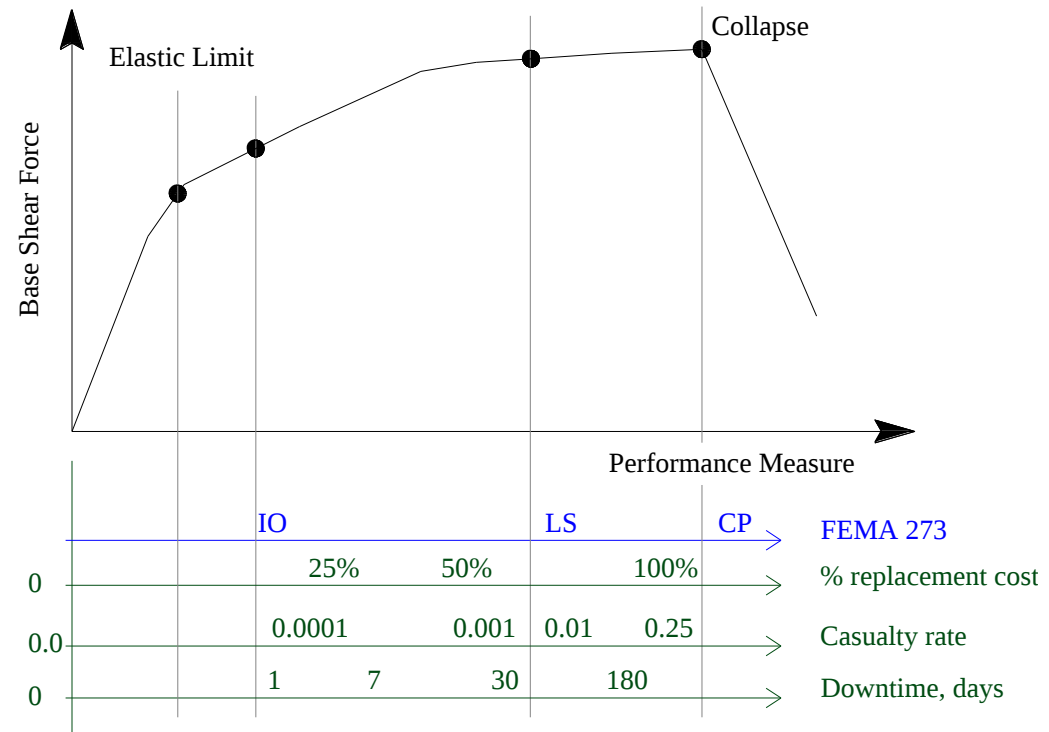
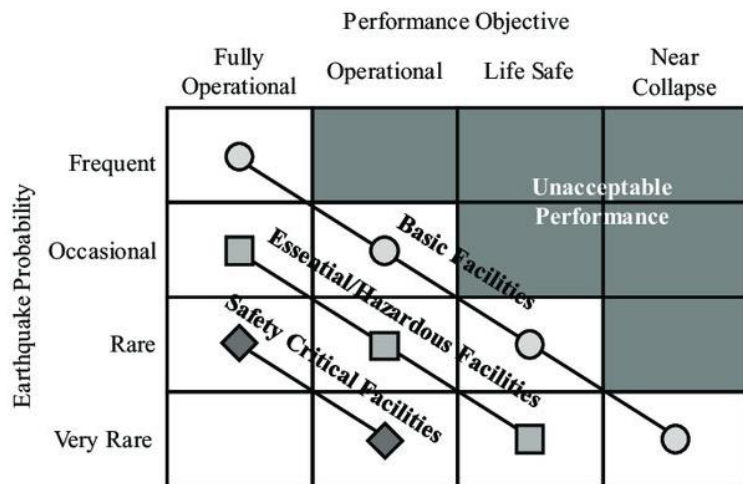
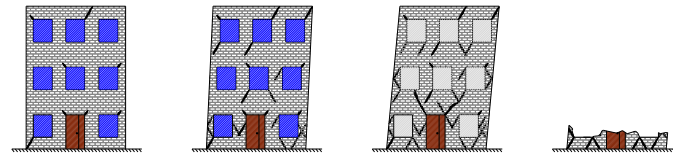
Earthquake Performance Levels



Intensity level ↓ +	Fully Operational	There is no damage to the structural and non-structural elements
	Operational	- Damages are very low - Non-structural element present low-damages - Structural elements are undamaged or can be easily repaired
	Life Safe	- The structure is damaged significantly - Non structural elements have damages - It is not convenient to repair the building
	Near Collapse	- The structure is strongly damaged - Almost all non-structural elements are damaged - The building can survive modest accelerations

Definition of the hazard

All the modern codes specify under seismic actions minimum levels of performance that structures have to fulfill.



Definition of the hazard

Hazard - Latvia


SNOW


WIND


SEISMIC

STANDARD

EN 1998-1

COUNTRY | ANNEX

Latvia | LVS EN 1998-1

SEISMIC LOAD | EN 1998-1 | LVS EN 1998-1

Enter a location

LOCATION

Street	Kaļķu iela 1
Postal Code	1050
City	Rīga
Latitude	56.947°
Longitude	24.104°
Altitude	4 m

a_{gR} in % of g

2

Acceleration of Gravity $g = 9.81 \text{ m/s}^2$

Reference Value of Peak Ground Acceleration

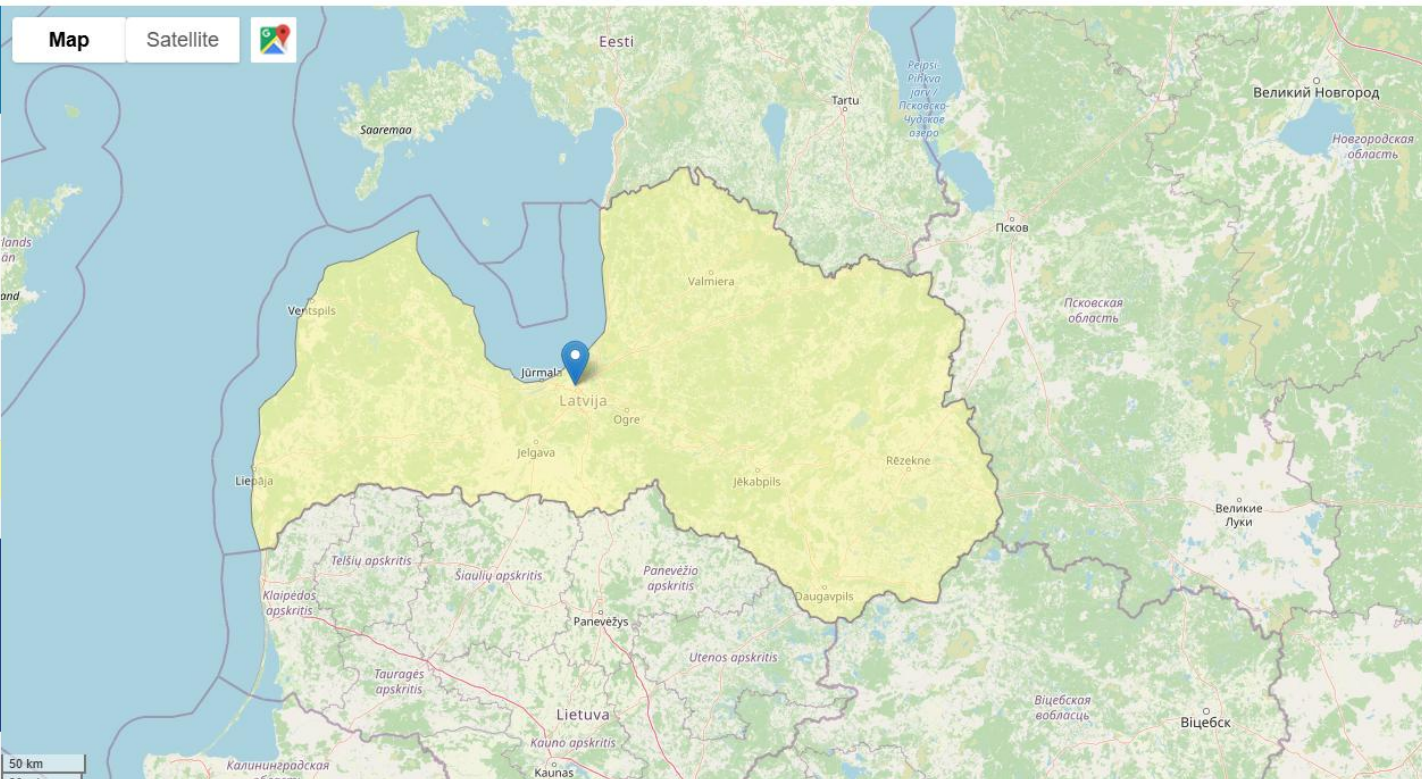
 $a_{gR} = 0.20 \text{ m/s}^2$

No liability for the data provided

 Map

 Satellite







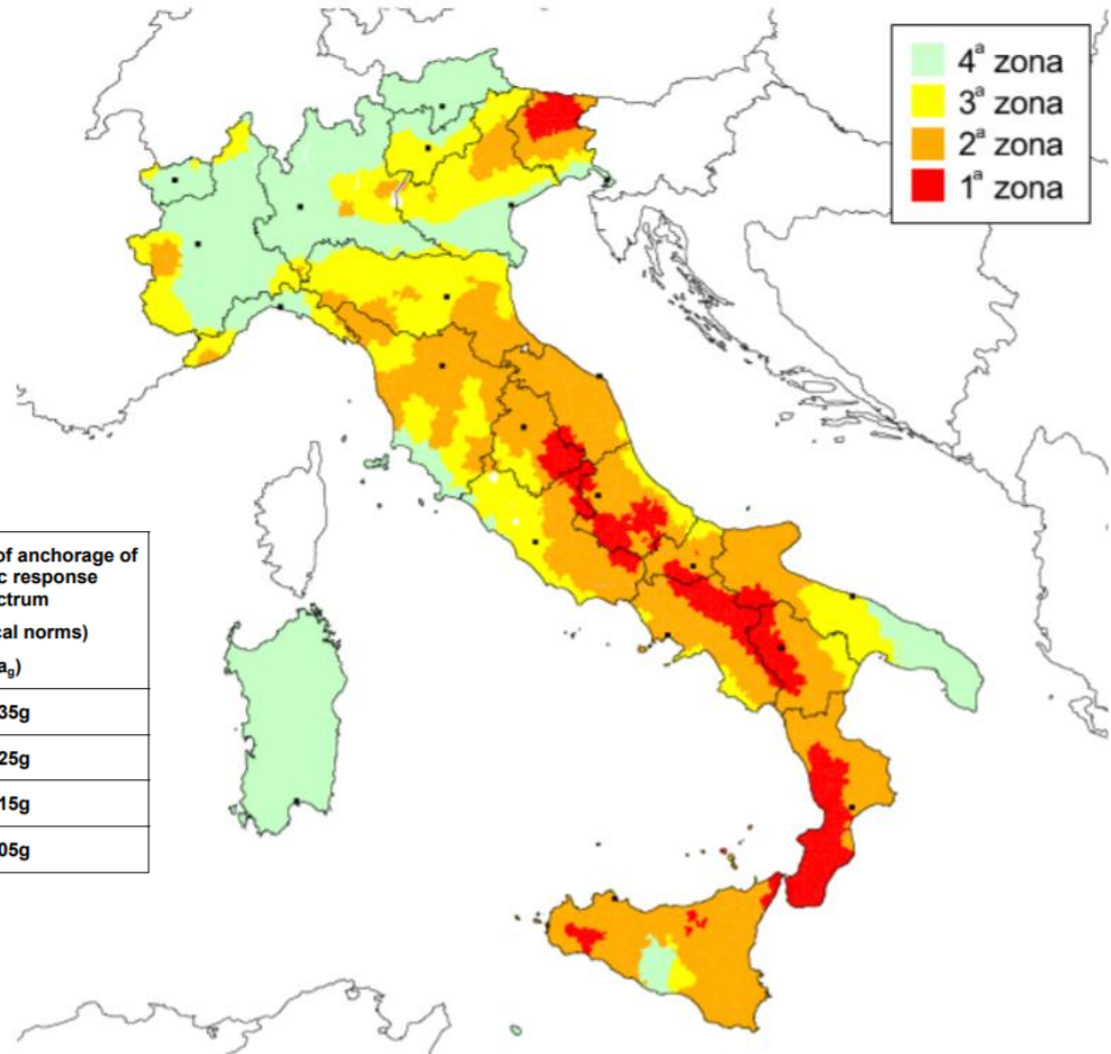






Definition of the hazard

Hazard - Italy

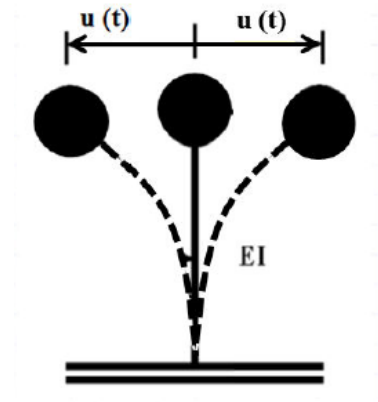


Seismic zone	Ground acceleration with probability of exceedance equal to 10% in 50 years (a_g)	Acceleration of anchorage of the elastic response spectrum (Technical norms) (a_g)
1	$> 0.25g$	0.35g
2	$0.15g - 0.25g$	0.25g
3	$0.05g - 0.15g$	0.15g
4	$< 0.05g$	0.05g

Principles of dynamics

Principles of dynamics

Degrees of Freedom (DoF): it is the number of independent variables needed to describe the motion of the system at any instant of time



SDoF: examples



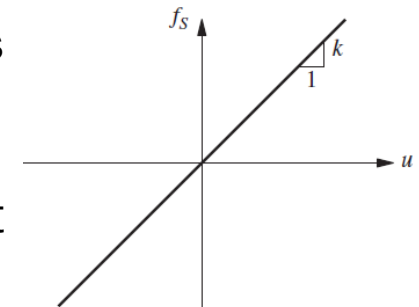
Principles of dynamics

SDoFs: resisting force f_s

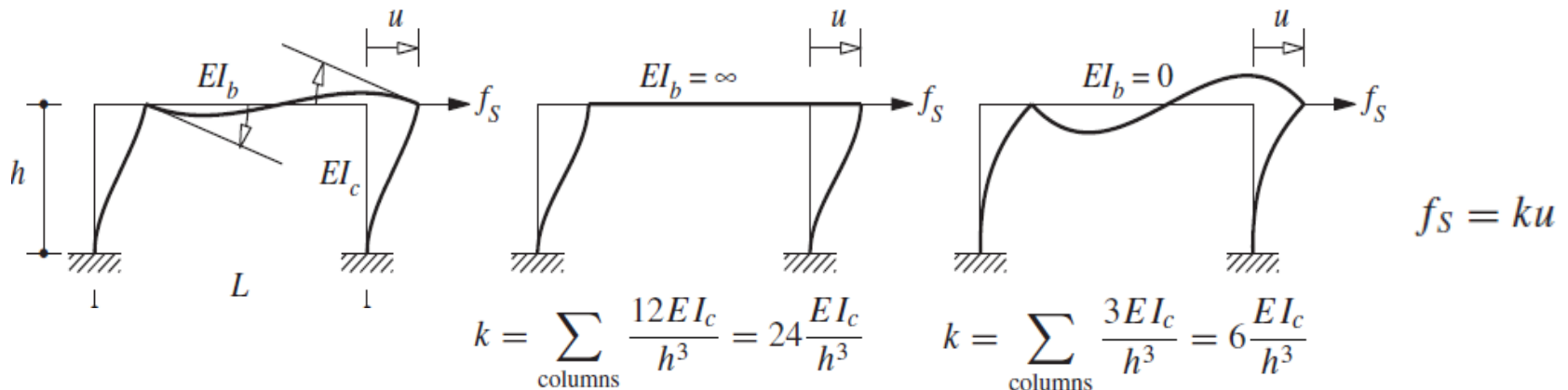
The resisting force (f_s) is the elastic force a structure uses to oppose displacement, and it depends on the system's stiffness.

Greater flexibility changes the system's dynamic behavior — it vibrates differently and responds differently to loads.

This is key in understanding the dynamic properties of buildings and mechanical systems.



(d)

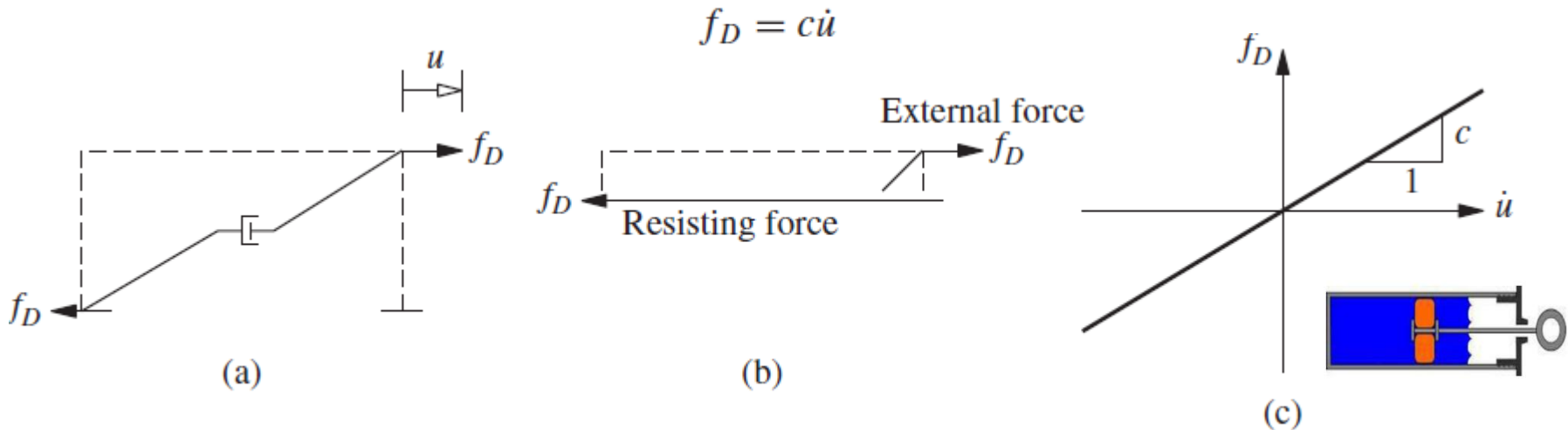


Principles of dynamics

SDoFs: damping force f_d

Damping is how a structure's vibrations slowly **fade away** over time. This happens because **energy is lost through things like friction, tiny cracks opening and closing, and interaction with air, fluids, or walls.**

Since these effects are complex, we usually model damping using a simple tool called a **dashpot**, which acts like a shock absorber.

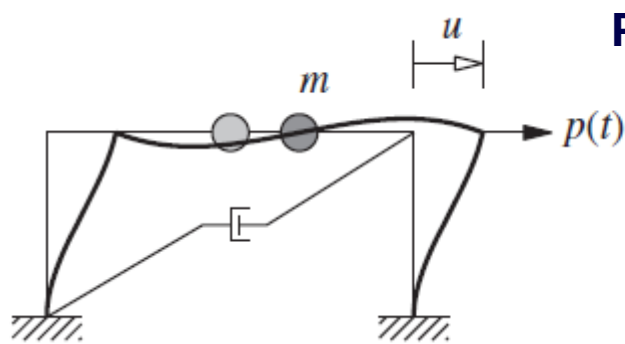


Principles of dynamics

Equation of motion for a SDoF: with a force applied

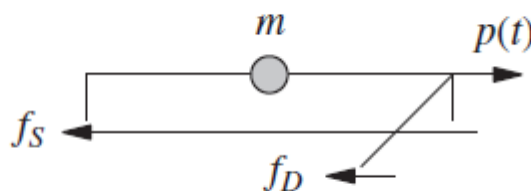
In dynamics, we can use **D'Alembert's Principle** to write the **equation of motion**. By adding the **inertia force** (a fictitious force), the system can be treated as if it's in **equilibrium at every moment** in time.

This allows us to apply **static-like equilibrium** to dynamic problems. It is an easy way to study motion using familiar equilibrium concepts.



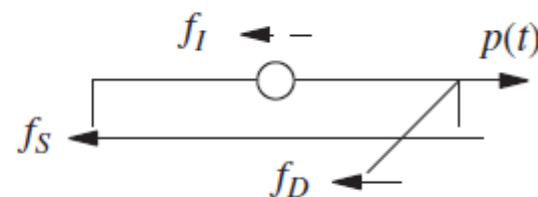
$$m\ddot{u} + f_D + f_s = p(t)$$

Free-body diagram



Elastic systems

$$m\ddot{u} + c\dot{u} + ku = p(t)$$

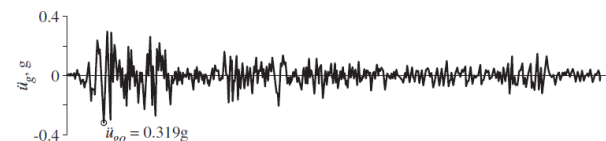
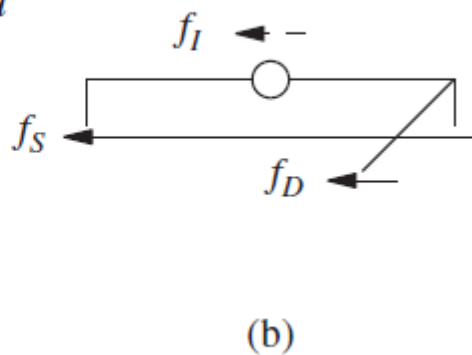
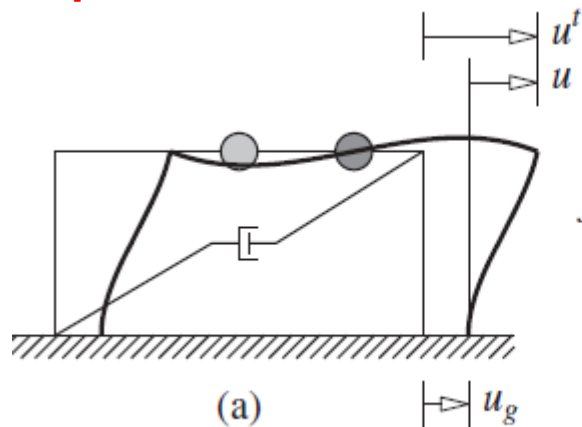


inelastic systems

$$m\ddot{u} + c\dot{u} + f_s(u) = p(t)$$

Principles of dynamics

Equation of motion for a SDoF: with earthquake excitation

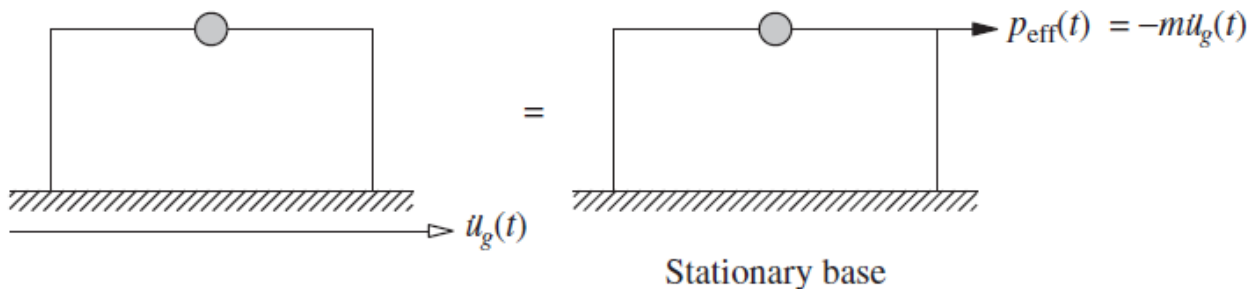


$$u^t(t) = u_g(t) + u(t)$$

Effective earthquake force

$$f_I + f_D + f_S = 0 \quad \xrightarrow{f_I = m\ddot{u}^t} \quad m\ddot{u} + c\dot{u} + ku = -m\ddot{u}_g(t)$$

$$m\ddot{u} + c\dot{u} + f_S(u) = -m\ddot{u}_g(t)$$



Principles of dynamics

Goal: Find the Response of a System

Force $m\ddot{u} + c\dot{u} + ku = p(t)$

Seismic $m\ddot{u} + c\dot{u} + ku = -m\ddot{u}_g(t)$

Given the system's **mass (m)**, **stiffness (k)**, **damping (c)**, and **dynamic excitation** (like an external force or ground motion), the main goal is to find how the system responds. We are interested in its:

- **Displacement**
- **Velocity**
- **Acceleration**

In earthquakes, we look at both **absolute** and **relative displacement**, but **relative displacement** is more important — it controls how the structure behaves.

SDoFs undergoing free vibration

Dynamic of SDoF systems

Free vibration of SDoFs: **undamped**

$$\cancel{m\ddot{u} + c\dot{u} + ku = p(t)}$$

(1) $m\ddot{u} + ku = 0$

Initial conditions

$$u = u(0) \quad \dot{u} = \dot{u}(0)$$

(2) $\delta^2 + \omega^2 = 0$ **where** $\omega_n = \sqrt{\frac{k}{m}}$

(3) $\delta = \pm i\omega$ **Remembering that**

$$y(t) = c_1 e^{\alpha t} \cos(\beta t) + c_2 e^{\alpha t} \sin(\beta t)$$

solution $u(t) = c_1 \cos(\omega t) + c_2 \sin(\omega t)$

$$u(t) = u(0) \cos \omega_n t + \frac{\dot{u}(0)}{\omega_n} \sin \omega_n t$$

Dynamic of SDoF systems

Free vibration of SDoFs: undamped

$$m\ddot{u} + c\dot{u} + ku = p(t)$$

Natural period of vibration

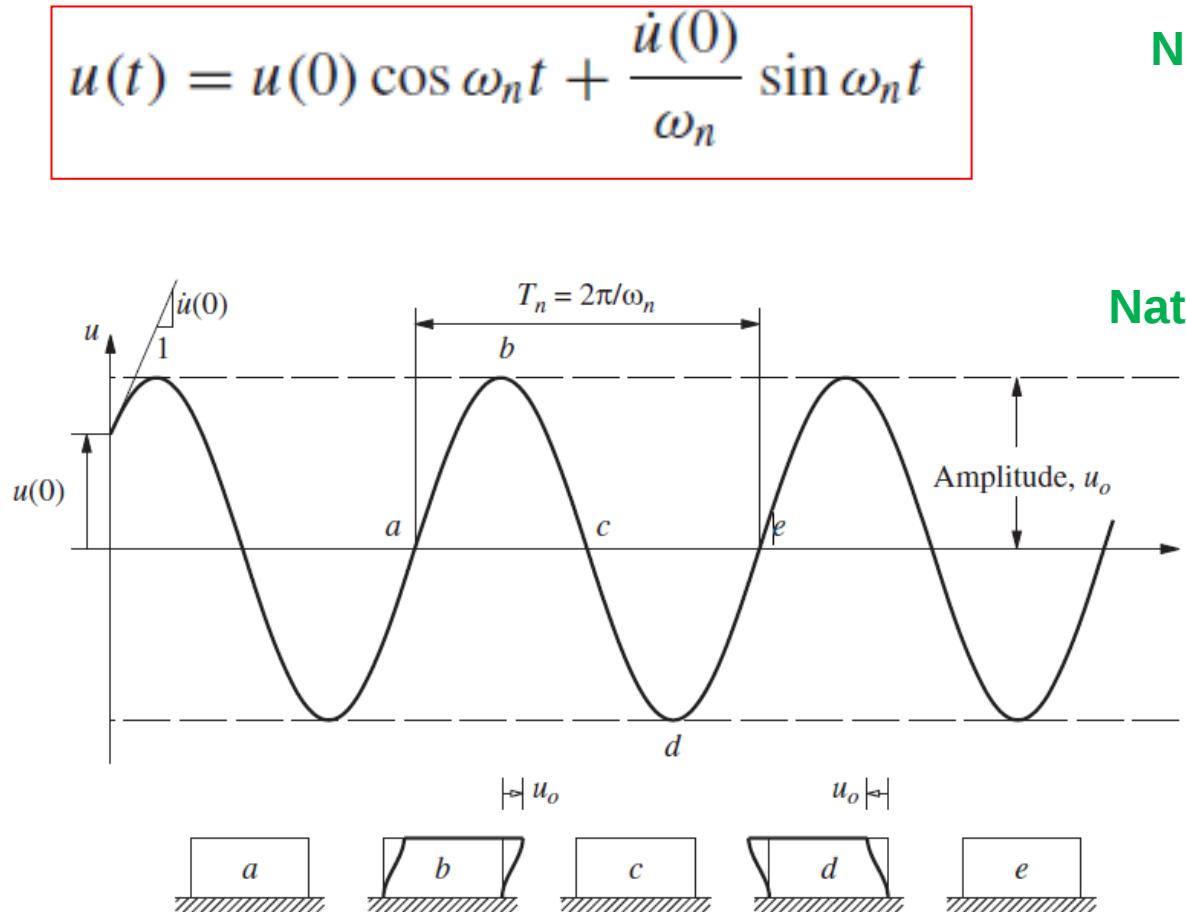
$$T_n = \frac{2\pi}{\omega_n} \quad \omega_n = \sqrt{\frac{k}{m}}$$

Natural frequency of vibration

$$f_n = \frac{1}{T_n}$$

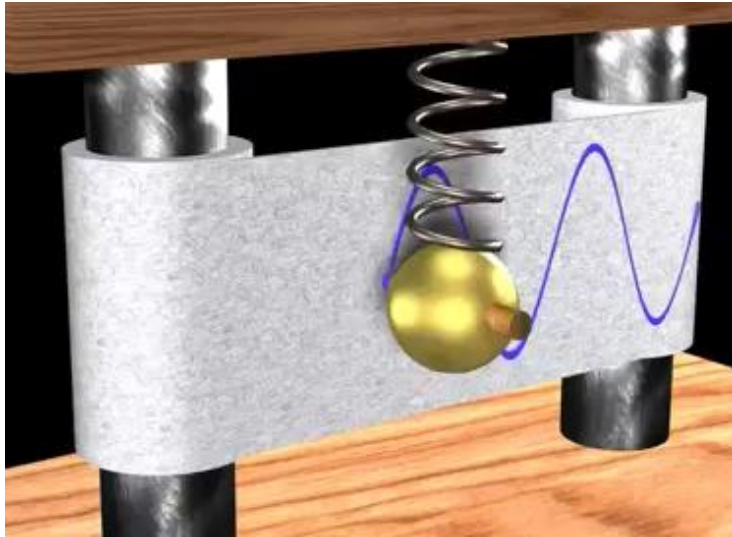
Amplitude

$$u_o = \sqrt{[u(0)]^2 + \left[\frac{\dot{u}(0)}{\omega_n}\right]^2}$$



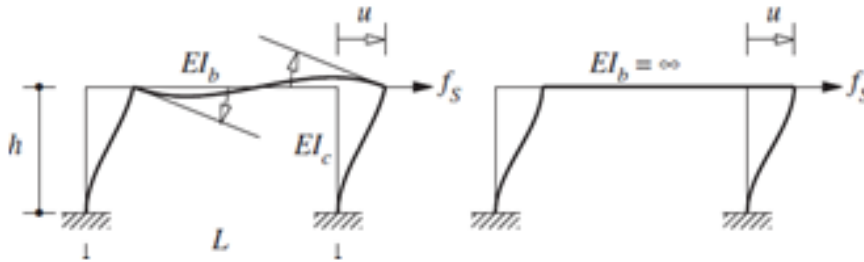
Dynamic of SDoF systems

Free vibration of SDoFs: **undamped**



Dynamic of SDoF systems

Free vibration of SDoFs: undamped



$$k = \sum_{\text{columns}} \frac{12EI_c}{h^3} = 24 \frac{EI_c}{h^3}$$

$$k = \frac{24EI_c}{h^3} = \frac{24 \times 210.000 \times 10^6 \times 14.920}{10^8 \times 3^3} = 27.850.667 \text{ N/m}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{27.850.667}{20.000}} = 37.31 \text{ Hz}$$

$$T_n = \frac{2\pi}{\omega_n} = 0.17 \text{ sec}$$

Data

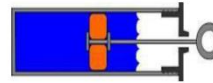
E=210.000 MPa

h=3 m I_c=14.920 cm⁴

m=20.000 kg

Dynamic of SDoF systems

Free vibration of SDoFs: damped



$$m\ddot{u} + c\dot{u} + ku = \cancel{p(t)}$$

(1) $m\ddot{u} + c\dot{u} + ku = 0$

where

$$\omega_n = \sqrt{\frac{k}{m}}$$

$$\zeta = \frac{c}{2m\omega_n} = \frac{c}{c_{cr}}$$

(2) $\ddot{u} + 2\zeta\omega_n\dot{u} + \omega_n^2 u = 0$

(3) $\lambda^2 + 2\zeta\omega_n\lambda + \omega_n^2 = 0$

(3) $\lambda_{1,2} = \omega_n(-\zeta \pm i\sqrt{1-\zeta^2})$ Remembering that

$$y(t) = c_1 e^{\alpha t} \cos(\beta t) + c_2 e^{\alpha t} \sin(\beta t)$$

**Solution
for $\zeta < 1$**

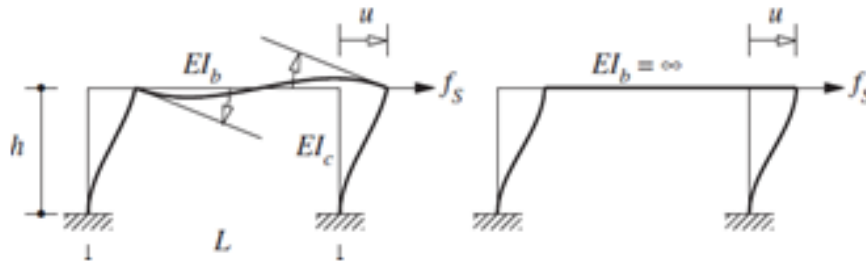
$$u(t) = e^{-\zeta\omega_n t} (A \cos \omega_D t + B \sin \omega_D t)$$

$$u(t) = e^{-\zeta\omega_n t} \left[u(0) \cos \omega_D t + \frac{\dot{u}(0) + \zeta\omega_n u(0)}{\omega_D} \sin \omega_D t \right]$$

$$\omega_D = \omega_n \sqrt{1 - \zeta^2}$$

Dynamic of SDoF systems

Free vibration of SDoFs: damped



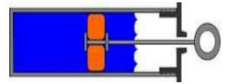
$$k = \sum_{\text{columns}} \frac{12EI_c}{h^3} = 24 \frac{EI_c}{h^3}$$

$$k = \frac{24EI_c}{h^3} = \frac{24 \times 210.000 \times 10^6 \times 14.920}{10^8 \times 3^3} = 27.850.667 \text{ N/m}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{27.850.667}{20.000}} = 37.31 \text{ Hz}$$

$$\omega_d = \omega_n \times \sqrt{1 - \zeta^2} = 37.27 \text{ Hz}$$

Data



E=210.000 MPa

h=3 m I_c=14.920 cm⁴

m=20.000 kg

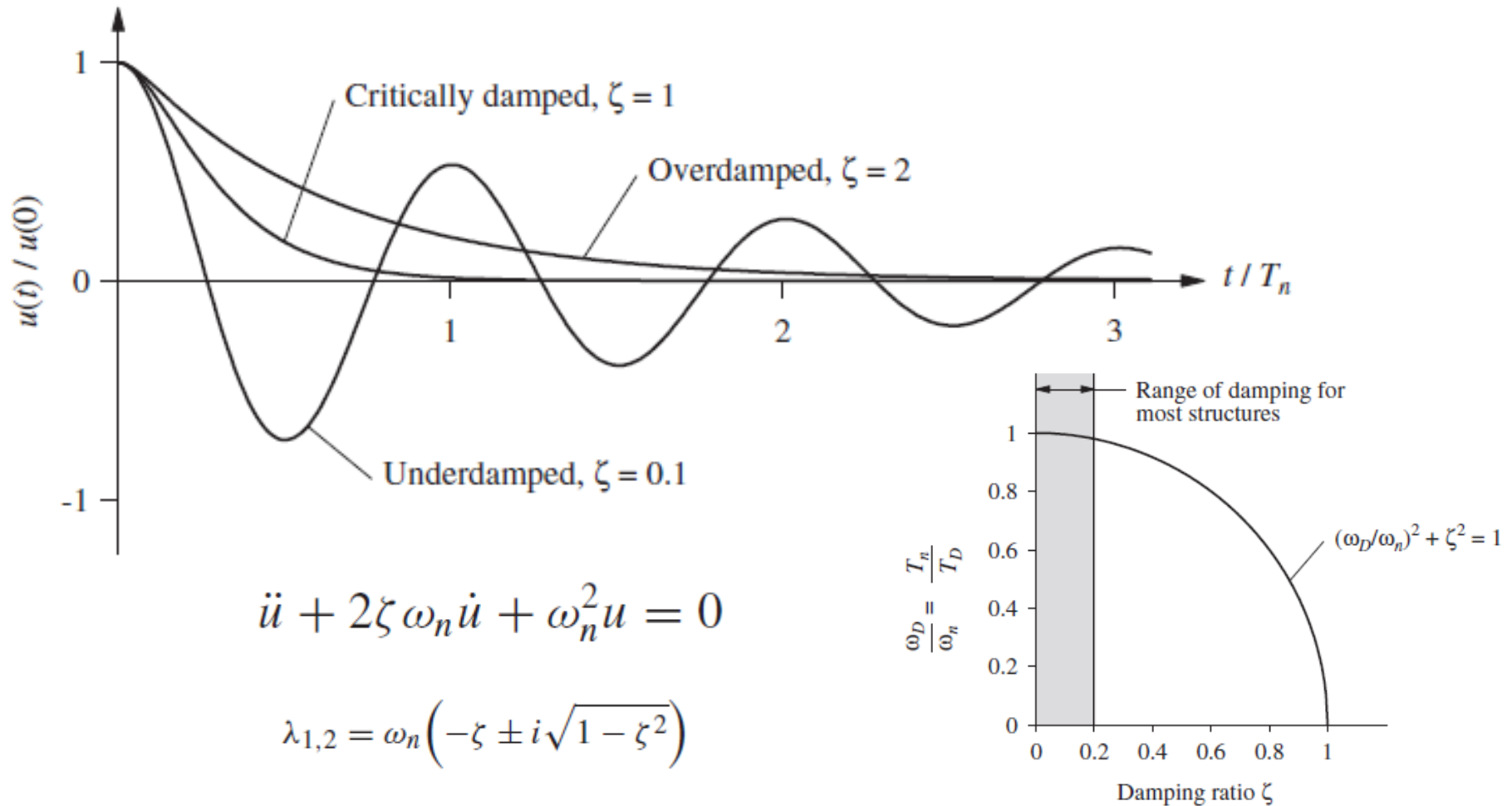
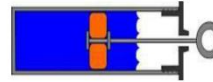
z=0.05

$$T_n = \frac{2\pi}{\omega_n} = 0.17 \text{ sec}$$

$$T_d = \frac{2\pi}{\omega_d} = 0.17 \text{ sec}$$

Dynamic of SDoF systems

Free vibration of SDoFs: **damped**



Dynamic of SDoF systems

Response of SDoFs: harmonic force, undamped

$$m\ddot{u} + \cancel{c\dot{u}} + ku = p(t)$$

(1) $m\ddot{u} + ku = p_o \sin \omega t$ where $p(t) = p_o \sin \omega t$

(2) $\delta^2 + \omega_n^2 = 0$ (3) $\omega_n = \sqrt{\frac{k}{m}}$

(3) $\delta = \pm i\omega_n$

Remembering that

$$y(t) = c_1 e^{\alpha t} \cos(\beta t) + c_2 e^{\alpha t} \sin(\beta t)$$

Solution

$$u(t) = A \cos \omega_n t + B \sin \omega_n t + \frac{p_o}{k} \frac{1}{1 - (\omega/\omega_n)^2} \sin \omega t$$

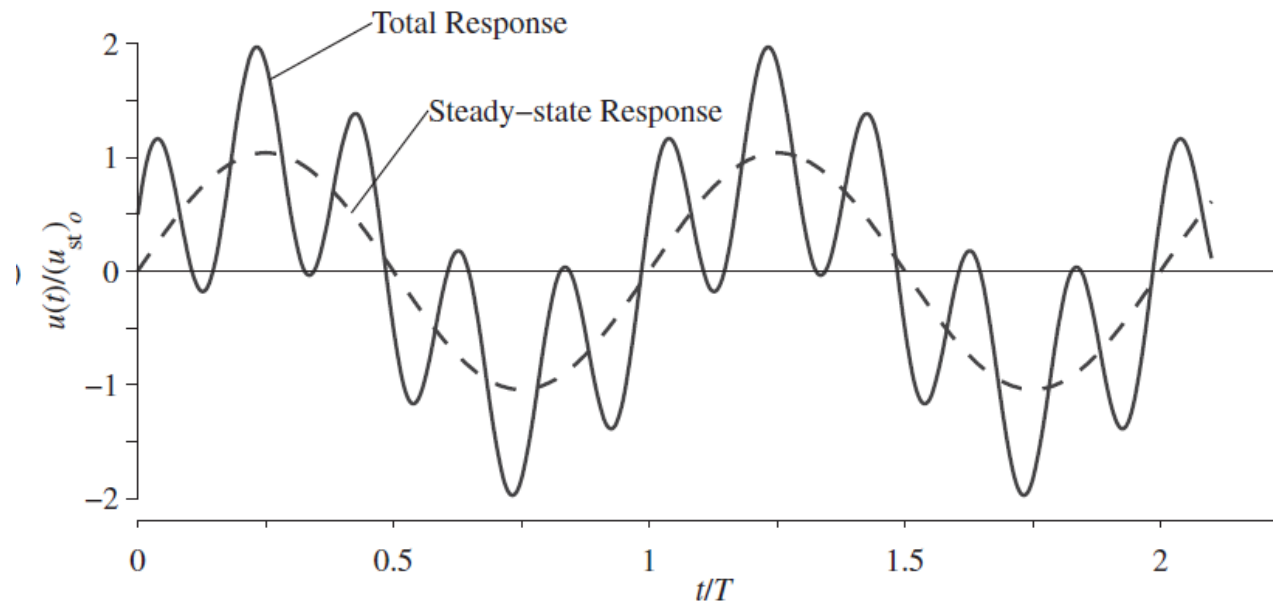
Dynamic of SDoF systems

Response of SDoFs: harmonic force, undamped

$$m\ddot{u} + \cancel{c\dot{u}} + ku = p(t)$$

Solution

$$u(t) = \underbrace{u(0) \cos \omega_n t + \left[\frac{\dot{u}(0)}{\omega_n} - \frac{p_o}{k} \frac{\omega/\omega_n}{1 - (\omega/\omega_n)^2} \right] \sin \omega_n t}_{\text{transient}} + \underbrace{\frac{p_o}{k} \frac{1}{1 - (\omega/\omega_n)^2} \sin \omega t}_{\text{steady state}}$$



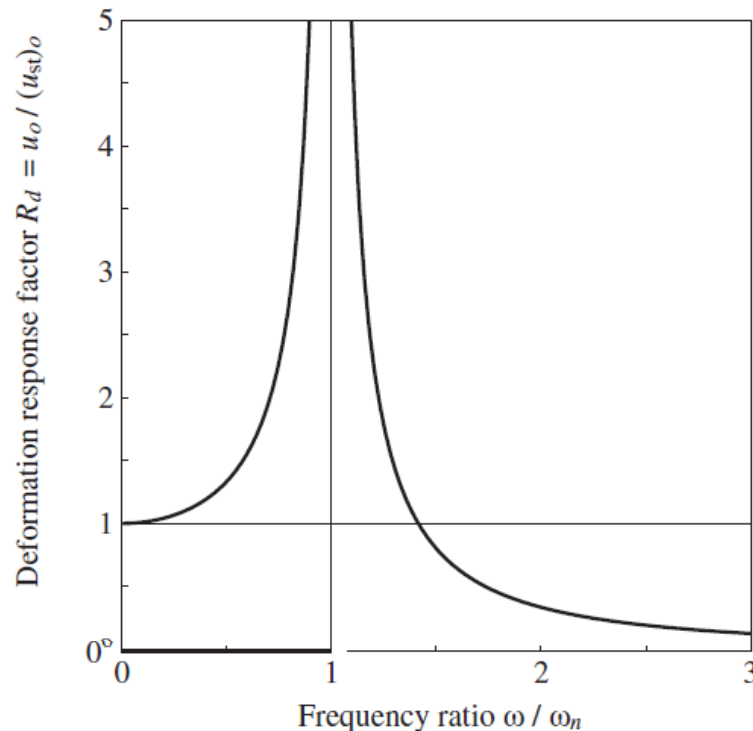
Dynamic of SDoF systems

Response of SDoFs: harmonic force, undamped

$$\cancel{m\ddot{u} + c\dot{u} + ku = p(t)}$$

$$u(t) = u_o \sin(\omega t - \phi) = (u_{st})_o R_d \sin(\omega t - \phi)$$

$$R_d = \frac{u_o}{(u_{st})_o} = \frac{1}{|1 - (\omega/\omega_n)^2|} \quad \text{and} \quad \phi = \begin{cases} 0^\circ & \omega < \omega_n \\ 180^\circ & \omega > \omega_n \end{cases}$$



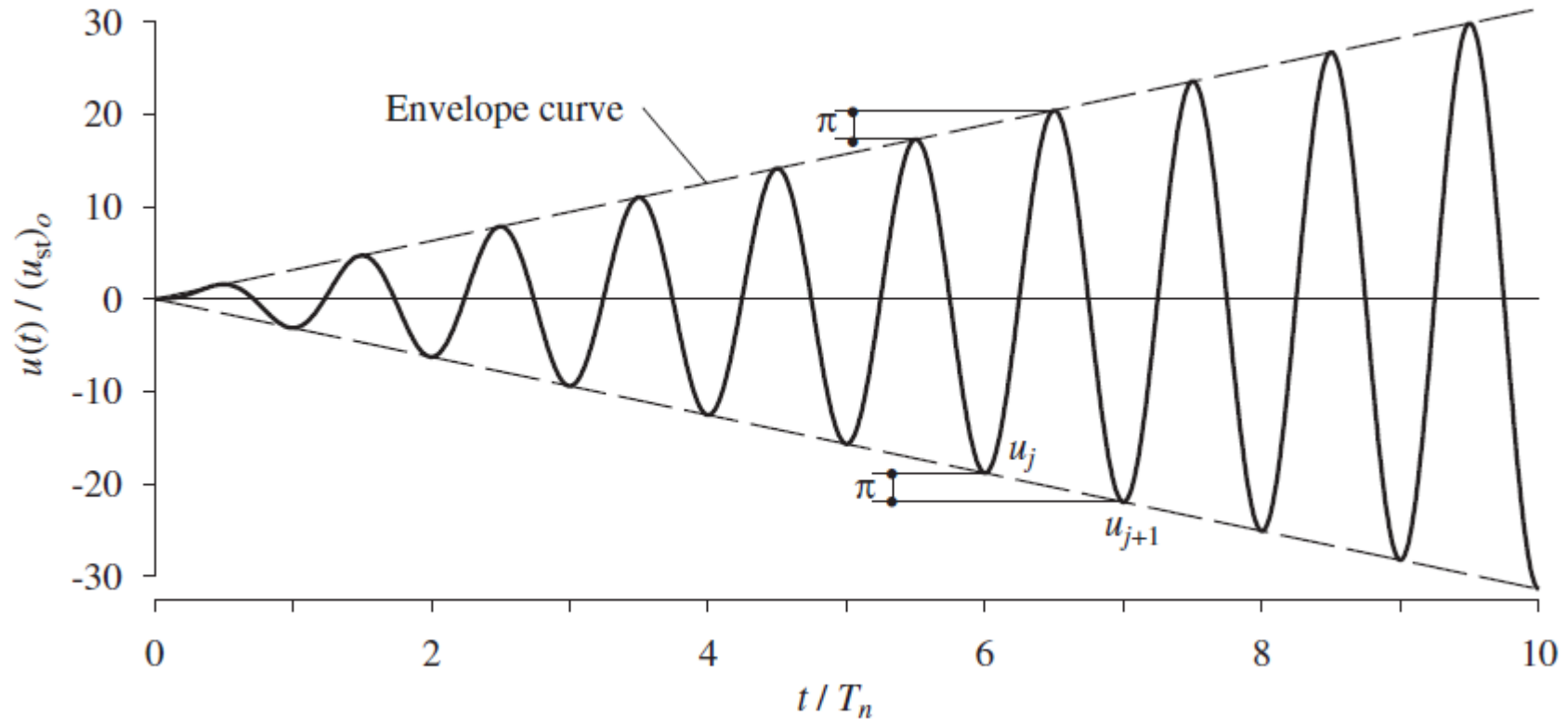
u_o is the amplitude of the steady-state response

ϕ is the phase angle (in-phase or out-of-phase)

Dynamic of SDoF systems

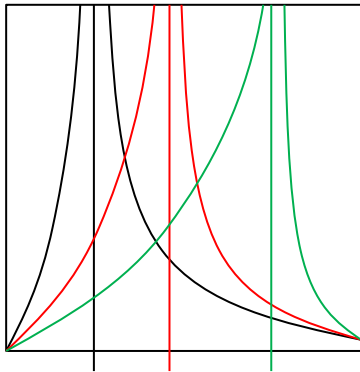
Response of SDoFs: harmonic force, undamped

$$m\ddot{u} + c\dot{u} + ku = p(t)$$



Dynamic of SDOF systems

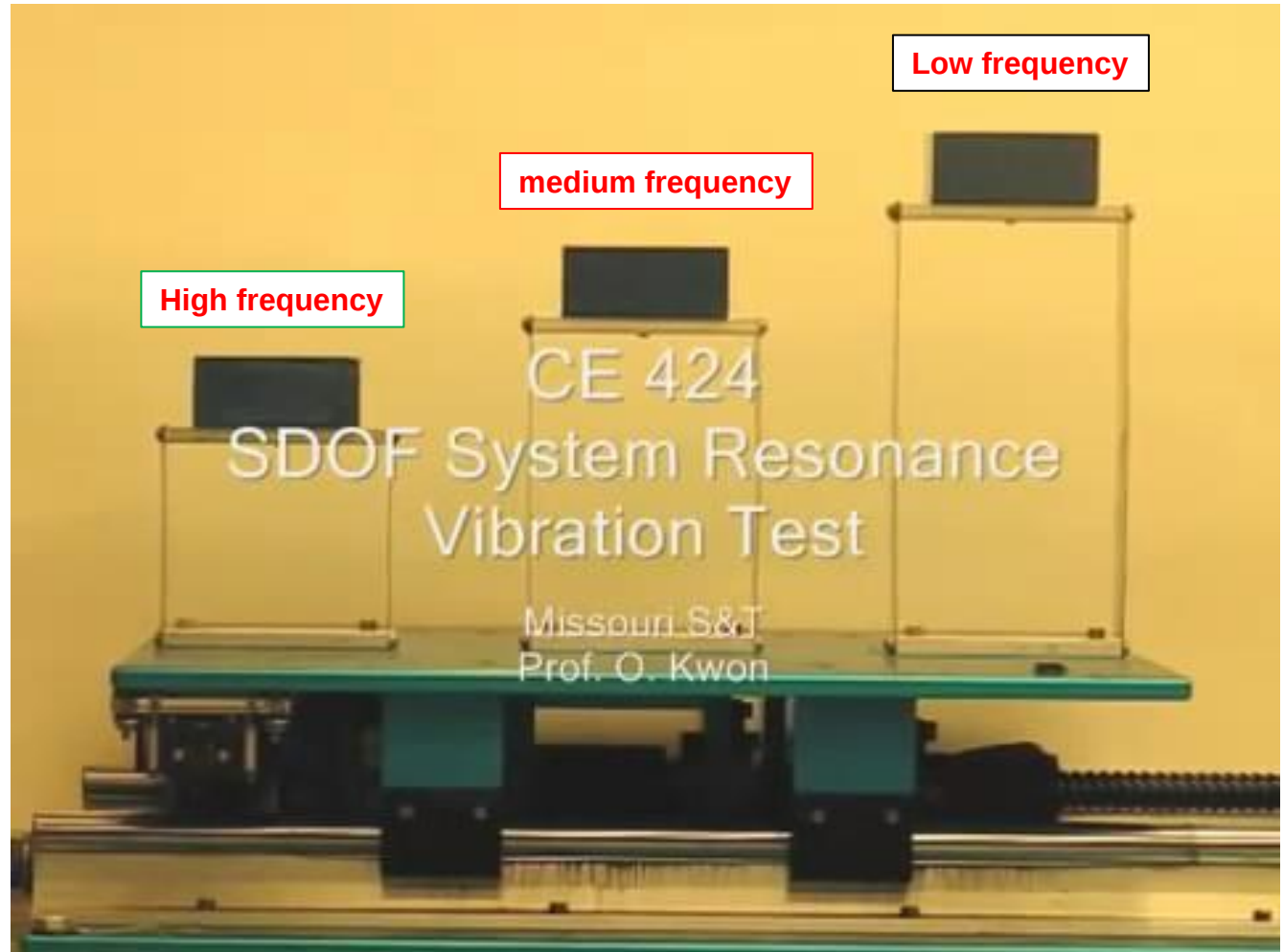
Response of SDOFs: harmonic force, undamped



Frequency ratio ω / ω_n

$$T_n = \frac{2\pi}{\omega_n}$$

$$\omega_n = \sqrt{\frac{k}{m}}$$



Dynamic of SDoF systems

Response of SDOFs: harmonic force, undamped



Dynamic of SDoF systems

Response of SDoFs: harmonic force, undamped



Seismic event Rieti (2016)

Dynamic of SDoF systems

Response of SDoFs: harmonic force, undamped



b)

Corso Umberto I - Amatrice

Effetti del sisma: a) Google street view pre-evento; b) immagine post-evento sismico.

Seismic event Rieti (2016)

Earthquake response of Linear systems

Earthquake response

1) The e.o.m. is solved by means of a numerical method (example central difference method)

$$\ddot{u} + 2\zeta\omega_n\dot{u} + \omega_n^2 u = -\ddot{u}_g(t)$$

2) The forces can be estimated from the response quantities;

$$f_D = c\dot{u} \quad \text{Damping force}$$

$$f_S(t) = ku(t) \quad \text{Restoring force}$$

$$f_I = m\ddot{u}^t \quad \text{Inertia force}$$

$$m\ddot{u} + c\dot{u} + ku = p(t) \quad \text{or} \quad -m\ddot{u}_g(t)$$

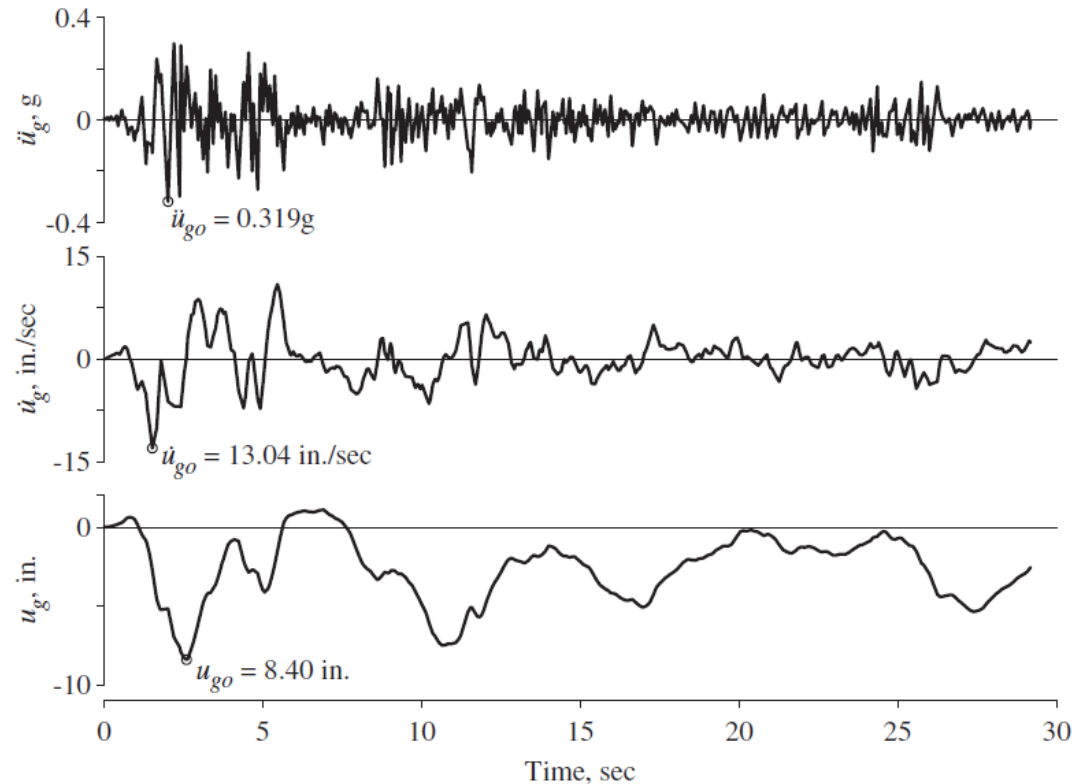


Figure 6.1.4 North-south component of horizontal ground acceleration recorded at the Imperial Valley Irrigation District substation, El Centro, California, during the Imperial Valley earthquake of May 18, 1940. The ground velocity and ground displacement were computed by integrating the ground acceleration.

Earthquake response of Linear systems

Equivalent force method

$$m\ddot{u} + c\dot{u} + ku = p(t) \quad \text{or} \quad -m\ddot{u}_g(t)$$

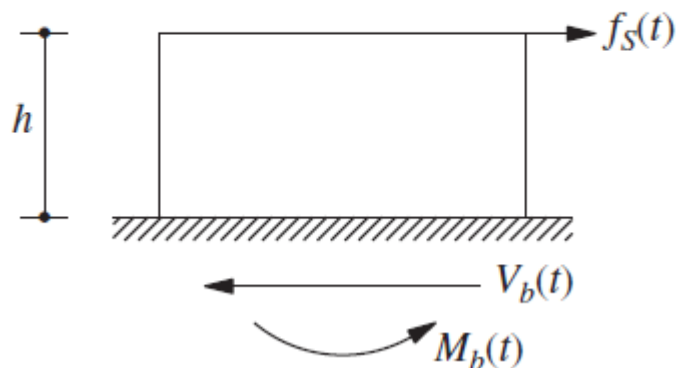
3) If $f_s = ku(t)$ is known, the stresses in the structure are known in each time instant (Moment, shear and axial force);

$$f_s(t) = ku(t)$$

$$f_s(t) = m \omega_n^2 u(t) = mA(t)$$

$$A(t) = \omega_n^2 u(t) \quad \text{Pseudo-acceleration}$$

$$V_b(t) = f_s(t) \quad M_b(t) = hf_s(t)$$



For a given earthquake, I can determine the relation between restoring force and mass through the pseudo-acceleration. This gives at any time instant the base shear and bending moment. We are interested in the peak response which is the one leading to the peak stresses.

Earthquake response of Linear systems

Basic Idea:

The **response spectrum** shows how **structures of different period (mass/stiffness)** respond to **the same earthquake**.

Think of it like testing how **all possible buildings** would shake if hit by the same ground motion.

Where it comes from:

Introduced by **Housner & Biot (1932)**

It summarizes the **maximum response** (displacement, velocity, or acceleration) of **single-degree-of-freedom systems (SDoF)**.

What it shows:

- **X-axis** = Period (T) → how flexible the structure is
 - **Y-axis** = Max response (acceleration, velocity, or displacement)
- The shape of the curve helps **engineers design safer buildings** by estimating the worst-case shaking

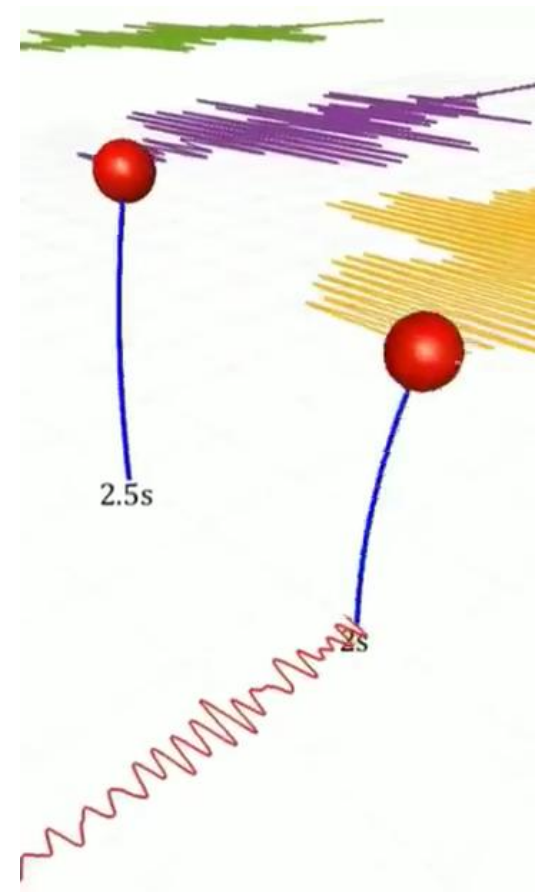
Why it matters:

Forms the **base of the lateral force method**

Used to estimate **design forces**

Helps turn **dynamic earthquake effects** into **simplified static loads**

Response spectrum

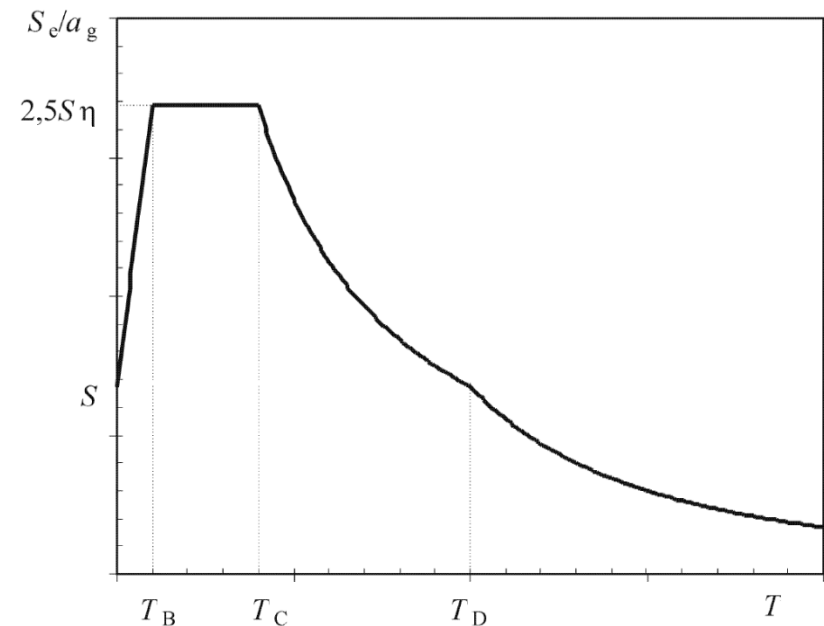
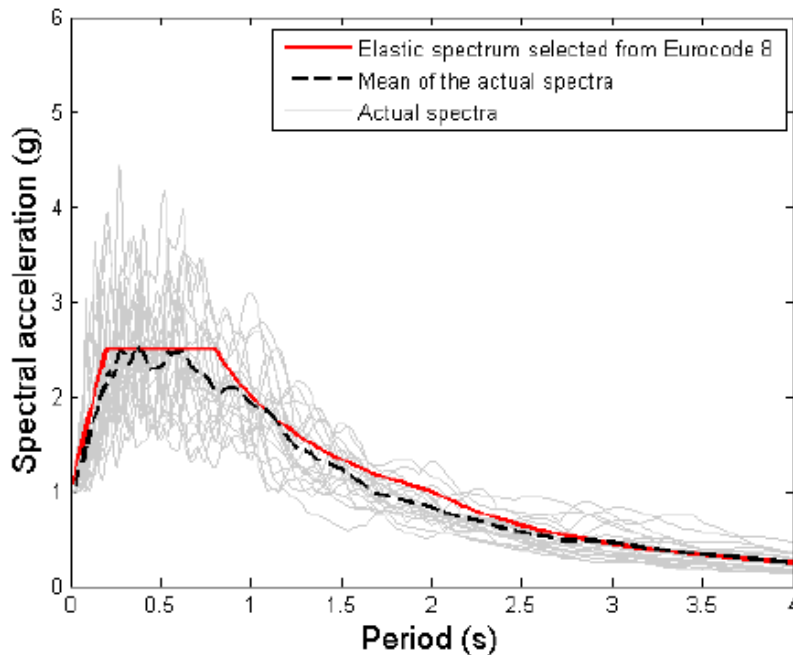


Earthquake response of Linear systems

EC8 Elastic design spectrum

Based on statistical analysis of past earthquake data at a given site, EC8 provides design spectra for both Serviceability Limit State (SLS) and Ultimate Limit State (ULS). The ULS spectrum is more severe, as it corresponds to a higher return period and lower probability of exceedance—in the Eurocodes 475 years for ULS and 95 years for SLS.

Seismic Elastic spectrum



Earthquake response of Linear systems

EC8 Elastic design spectrum

3.2.2.2 Horizontal elastic response spectrum

(1)P For the horizontal components of the seismic action, the elastic response spectrum $S_e(T)$ is defined by the following expressions (see Figure. 3.1):

$$0 \leq T \leq T_B : S_e(T) = a_g \cdot S \cdot \left[1 + \frac{T}{T_B} \cdot (\eta \cdot 2,5 - 1) \right]$$

Initially increases

$$T_B \leq T \leq T_C : S_e(T) = a_g \cdot S \cdot \eta \cdot 2,5$$

Constant Pseudo-Acceleration

$$T_C \leq T \leq T_D : S_e(T) = a_g \cdot S \cdot \eta \cdot 2,5 \left[\frac{T_C}{T} \right]$$

Constant Pseudo-velocity

$$T_D \leq T \leq 4s : S_e(T) = a_g \cdot S \cdot \eta \cdot 2,5 \left[\frac{T_C T_D}{T^2} \right]$$

Constant Displacement

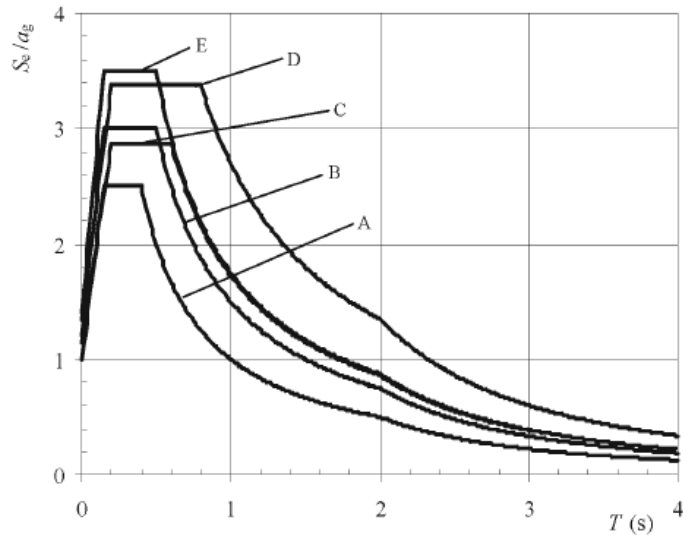
a_g is the PGA

T is natural period

$$\eta = \sqrt{10 / (5 + \xi)} \geq 0,55$$

S is a soil factor (classes from A-E) S_e is the pseudo-acceleration

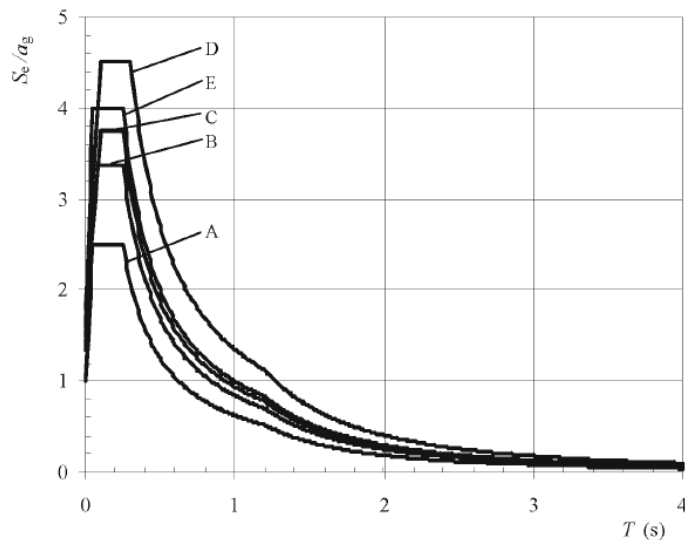
Earthquake response of Linear systems



$M_s > 5.5$ (High seismicity areas)

Table 3.2: Values of the parameters describing the recommended Type 1 elastic response spectra

Ground type	S	T_B (s)	T_C (s)	T_D (s)
A	1,0	0,15	0,4	2,0
B	1,2	0,15	0,5	2,0
C	1,15	0,20	0,6	2,0
D	1,35	0,20	0,8	2,0
E	1,4	0,15	0,5	2,0



$M_s < 5.5$ (Low seismicity areas)

Table 3.3: Values of the parameters describing the recommended Type 2 elastic response spectra

Ground type	S	T_B (s)	T_C (s)	T_D (s)
A	1,0	0,05	0,25	1,2
B	1,35	0,05	0,25	1,2
C	1,5	0,10	0,25	1,2
D	1,8	0,10	0,30	1,2
E	1,6	0,05	0,25	1,2

Earthquake response of Linear systems

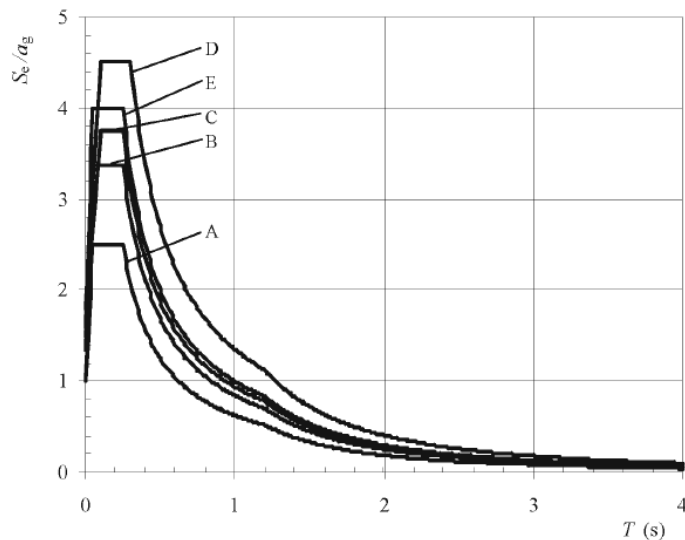
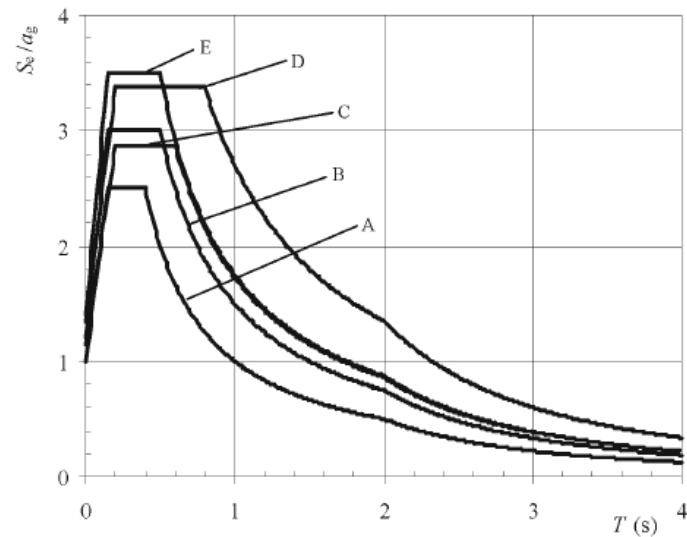


Table 3.1: Ground types

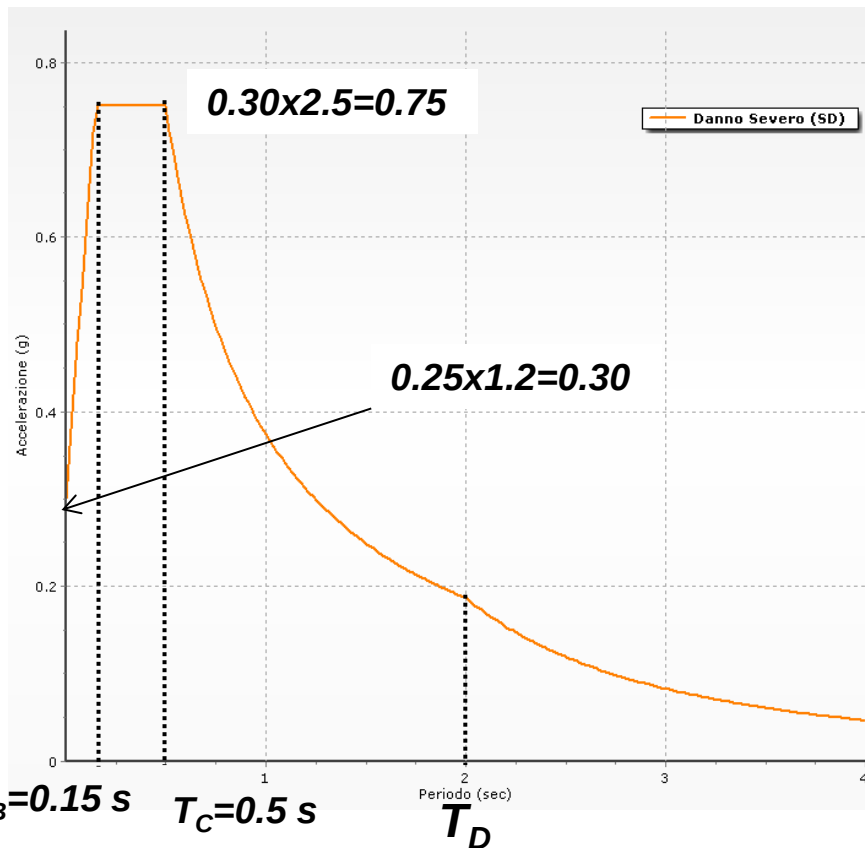
Ground type	Description of stratigraphic profile	Parameters		
		$v_{s,30}$ (m/s)	N_{SPT} (blows/30cm)	c_u (kPa)
A	Rock or other rock-like geological formation, including at most 5 m of weaker material at the surface.	> 800	—	—
B	Deposits of very dense sand, gravel, or very stiff clay, at least several tens of metres in thickness, characterised by a gradual increase of mechanical properties with depth.	$360 - 800$	> 50	> 250
C	Deep deposits of dense or medium-dense sand, gravel or stiff clay with thickness from several tens to many hundreds of metres.	$180 - 360$	$15 - 50$	$70 - 250$
D	Deposits of loose-to-medium cohesionless soil (with or without some soft cohesive layers), or of predominantly soft-to-firm cohesive soil.	< 180	< 15	< 70
E	A soil profile consisting of a surface alluvium layer with v_s values of type C or D and thickness varying between about 5 m and 20 m, underlain by stiffer material with $v_s > 800$ m/s.			
S_1	Deposits consisting, or containing a layer at least 10 m thick, of soft clays/silts with a high plasticity index ($PI > 40$) and high water content	< 100 (indicative)	—	$10 - 20$
S_2	Deposits of liquefiable soils, of sensitive clays, or any other soil profile not included in types A – E or S_1			

(2) The site should be classified according to the value of the average shear wave velocity, $v_{s,30}$, if this is available. Otherwise the value of N_{SPT} should be used.

Earthquake response of Linear systems

EC8 Elastic design spectrum - example

$a_g=0.25g$ – medium hazard $S=1.20$ – soil type B (hard soil)



$$0 \leq T \leq T_B : S_e(T) = a_g \cdot S \cdot \left[1 + \frac{T}{T_B} \cdot (\eta \cdot 2,5 - 1) \right]$$

$$T_B \leq T \leq T_C : S_e(T) = a_g \cdot S \cdot \eta \cdot 2,5$$

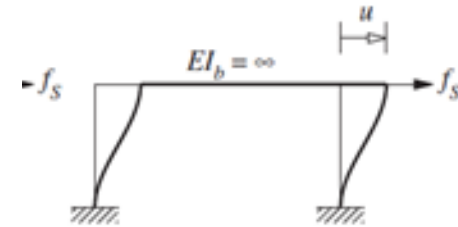
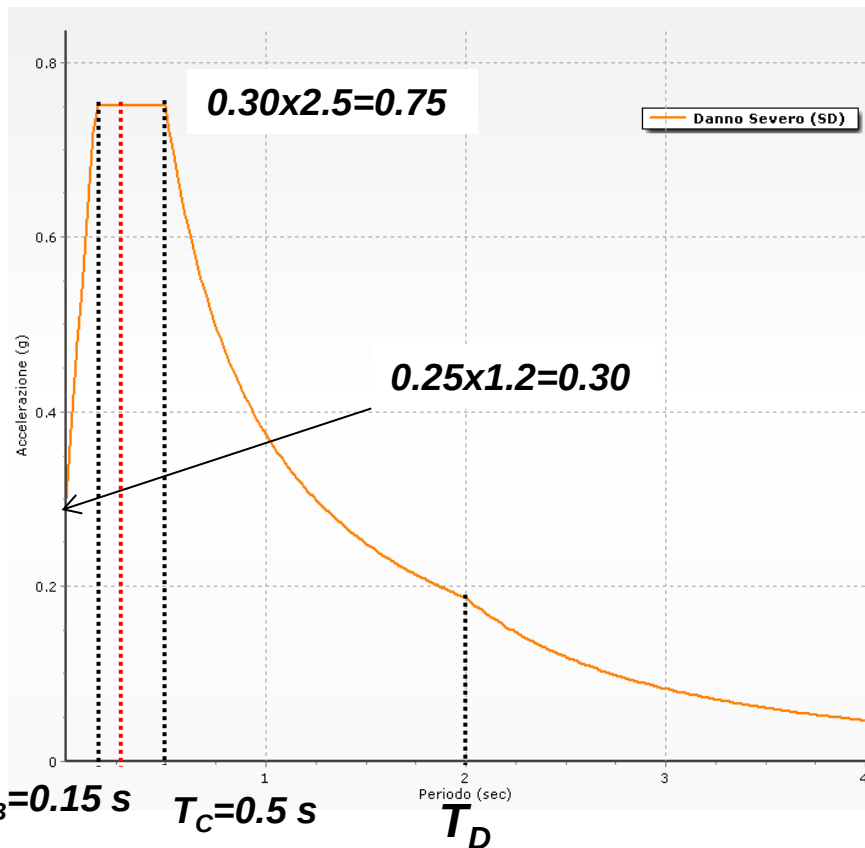
$$T_C \leq T \leq T_D : S_e(T) = a_g \cdot S \cdot \eta \cdot 2,5 \left[\frac{T_C}{T} \right]$$

$$T_D \leq T \leq 4s : S_e(T) = a_g \cdot S \cdot \eta \cdot 2,5 \left[\frac{T_C T_D}{T^2} \right]$$

Earthquake response of Linear systems

EC8 Elastic design spectrum - example

$a_g=0.25g$ – medium hazard $S=1.20$ – soil type B (hard soil)



$$k = \sum_{\text{columns}} \frac{12EI_c}{h^3} = 24 \frac{EI_c}{h^3}$$

$E=210.000$ MPa $m=20.000$ kg

$h=3$ m $I_c=14.920$ cm⁴

$$T_n = 2\pi \sqrt{\frac{m}{k}} = \sqrt{\frac{20.000}{27.850.667}} = 0.17 \text{ s}$$

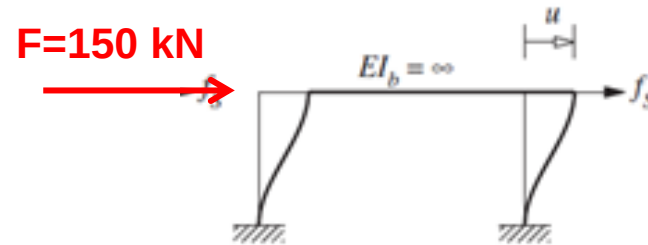
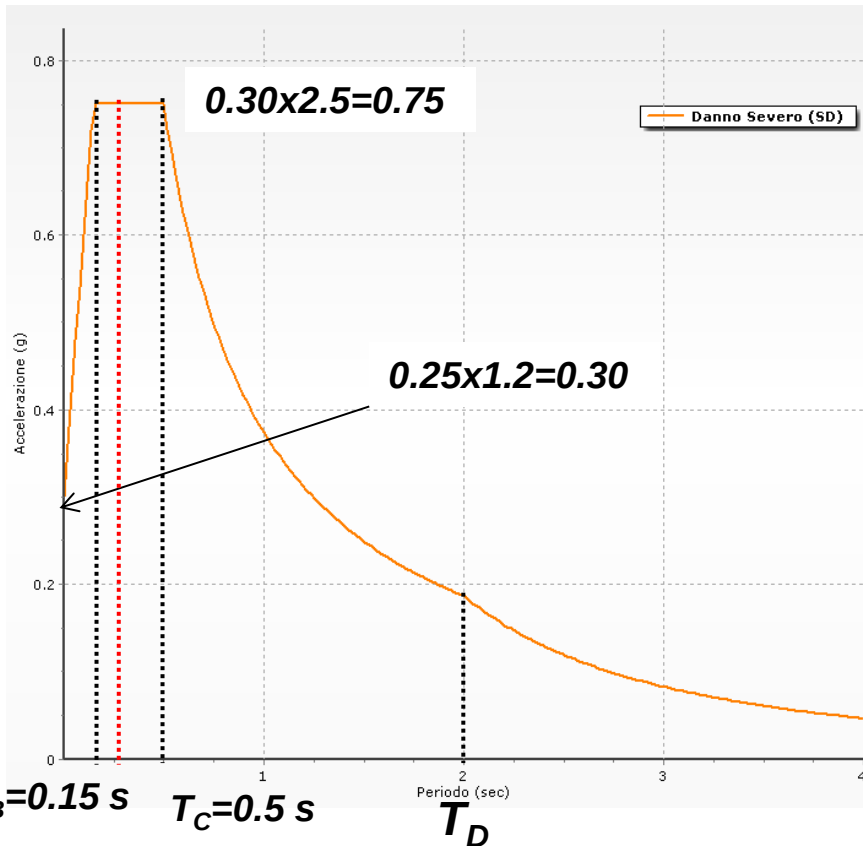
$$\omega_n = 37.31 \text{ Hz}$$

Earthquake response of Linear systems

EC8 Elastic design spectrum - example

$a_g=0.25g$ – medium hazard $S=1.20$ – soil type B (hard soil)

$$V_{bo} = f_{so} = mA$$



$$k = \sum_{\text{columns}} \frac{12EI_c}{h^3} = 24 \frac{EI_c}{h^3}$$

$$V_b = w \frac{S_e}{g} = 200.000 \times 0.75 = 150.000\text{ N}$$

The role of ductility

The role of ductility

This image shows how an **earthquake input energy** is absorbed and transformed by a structure:

Rain = Seismic Input Energy

The system (the "building") collects this energy, like rain through a **retractable roof**.

The energy is then "pumped" in the form of:

Kinetic Energy from ground motion which is transformed into:

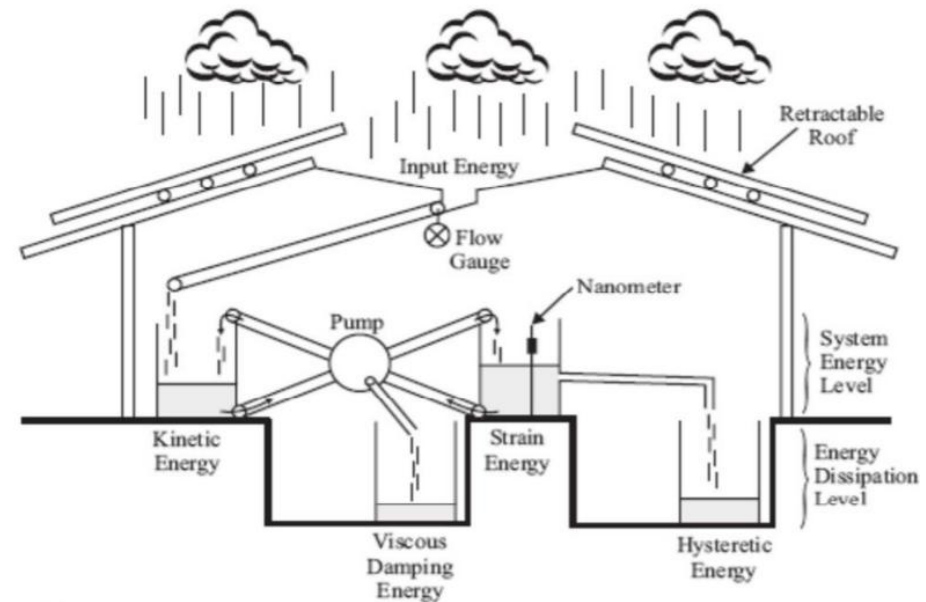
Strain Energy (stored in the structure) and **Viscous Damping Energy** (lost due to damping)

If **Strain Energy** exceeds the material's elastic limit, part of it is converted into **Hysteretic Energy**, which is **irreversibly lost** due to internal damage or plastic deformation.

The total **System Energy Level** shows how much energy the structure takes in.

The **Energy Dissipation Level** shows how much of that energy is safely absorbed or lost — critical for avoiding damage.

$$E_I(t) = E_k(t) + E_{vd}(t) + E_{es}(t) + E_h(t)$$



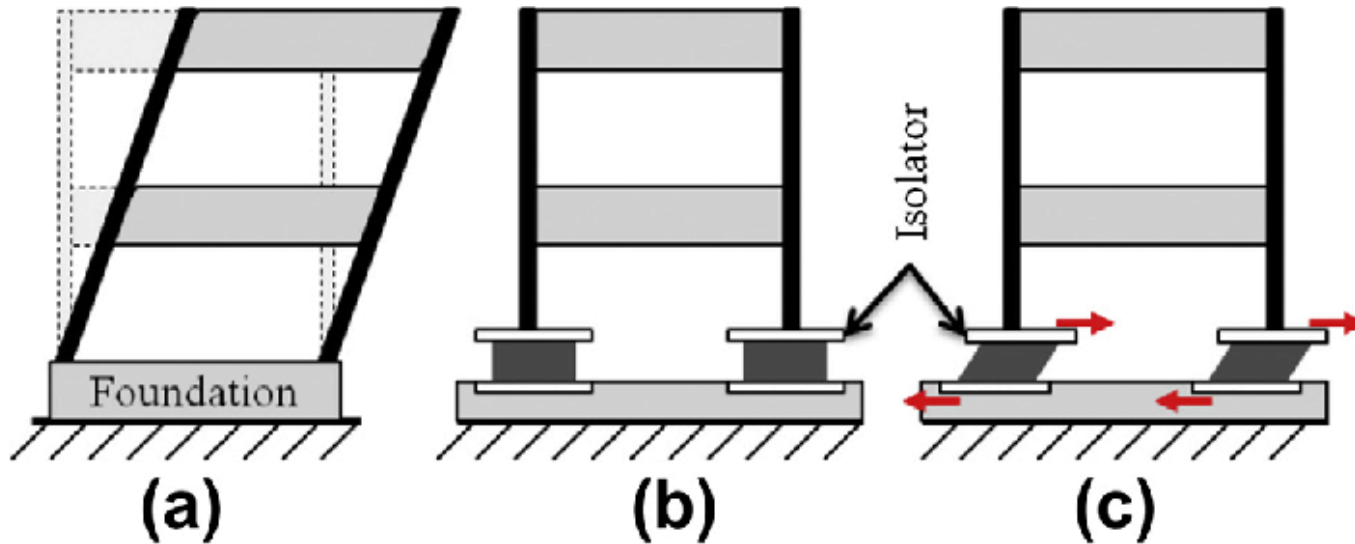
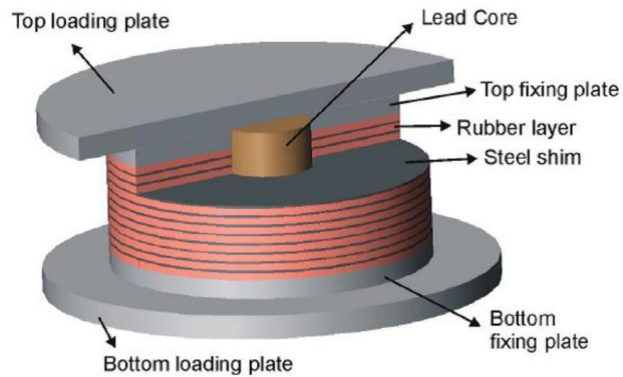
This analogy allows us to individuate the strategies for seismic protection

The role of ductility

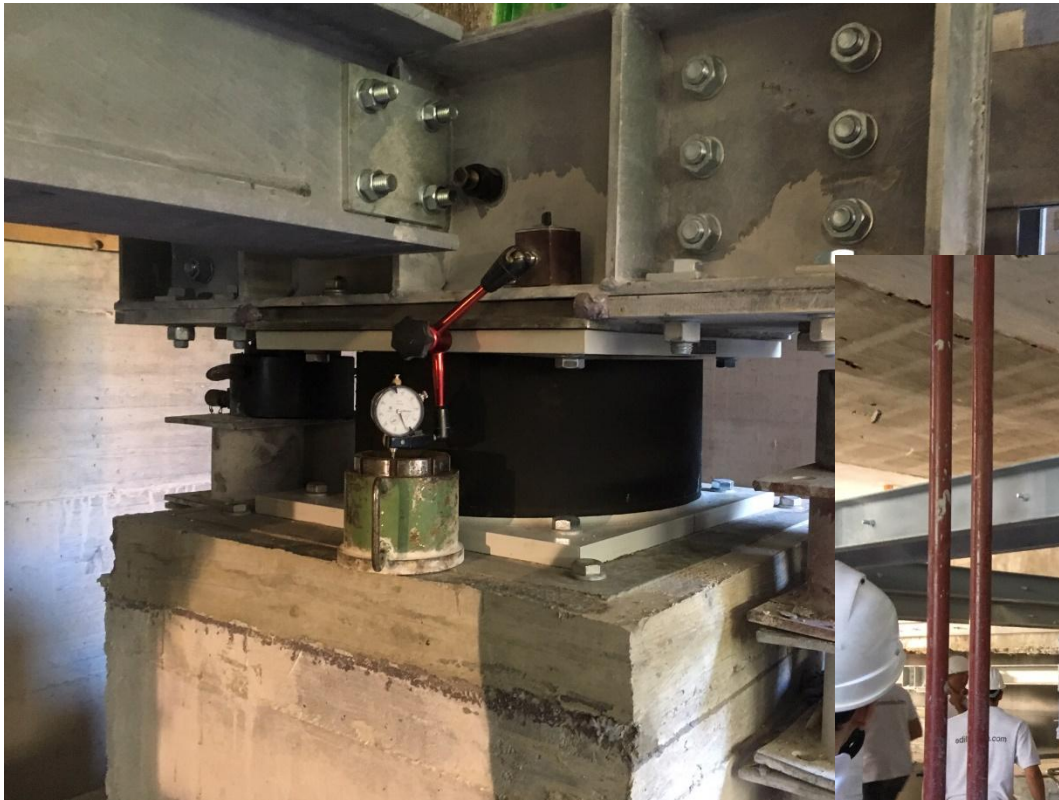
1st strategy: remove the input energy (close the roof)



The role of ductility



The role of ductility



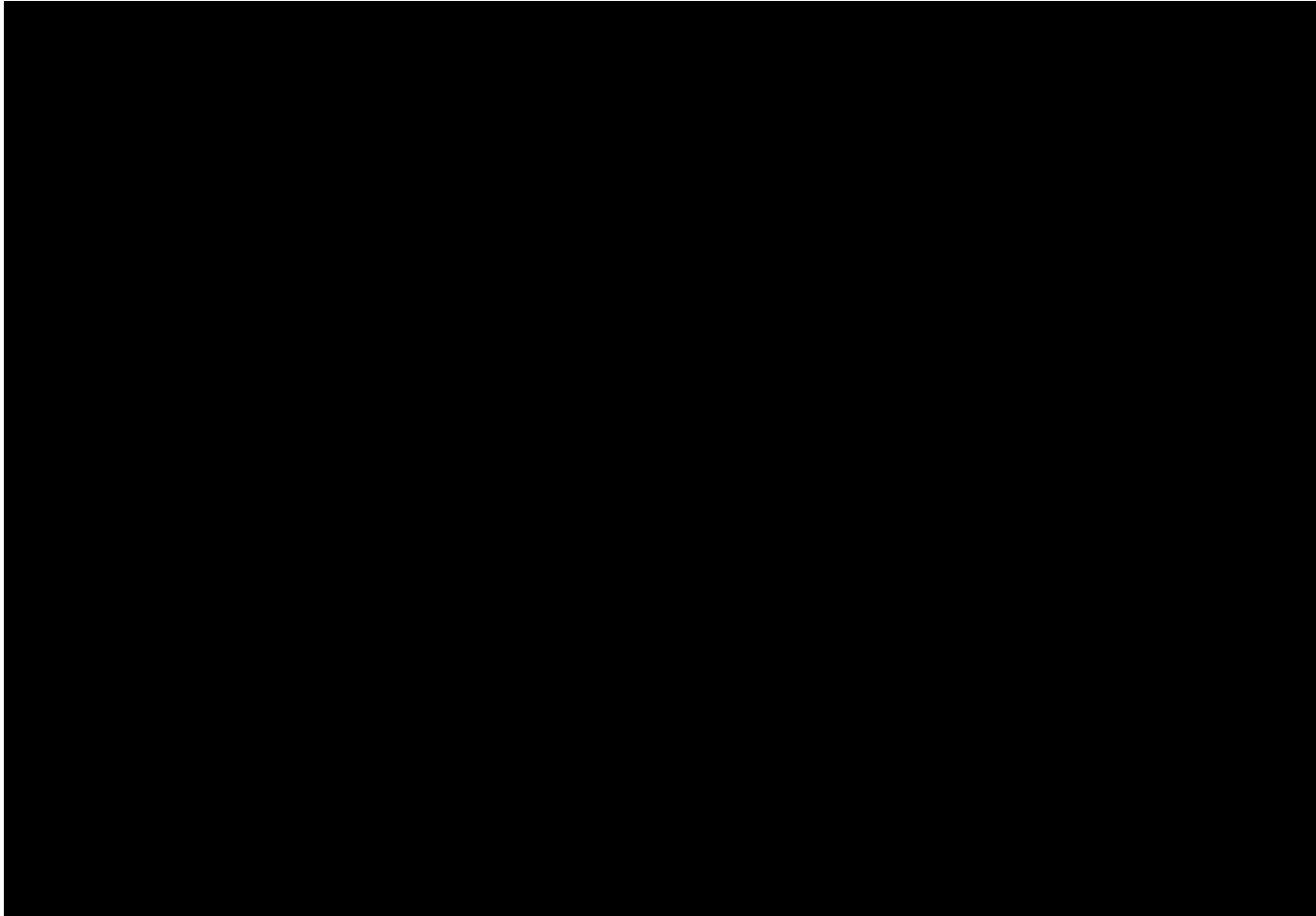
The role of ductility

2nd strategy: increase structural damping (increase energy lost in the pump)



The role of ductility

3rd strategy: modify the dynamic properties of the building (Tuned Mass Damper)



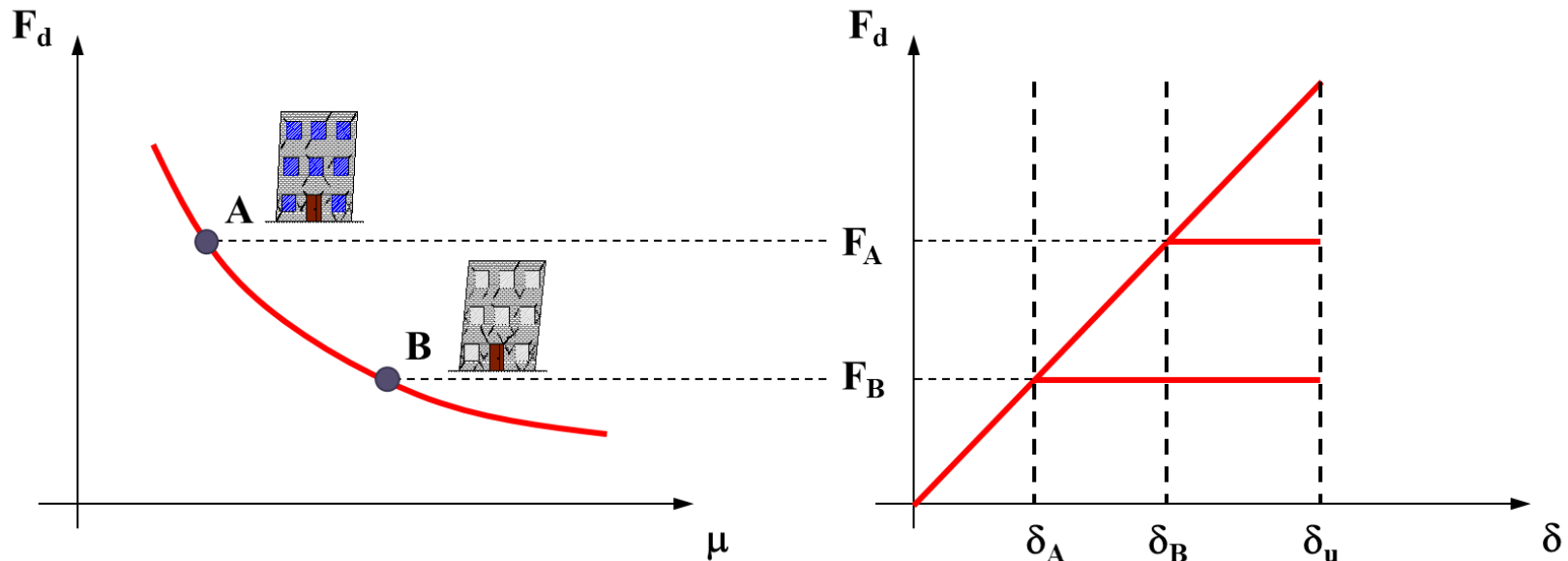
Taiwan

508 m

The role of ductility

The most common type of passive control: increase hysteresis

There is a relationship between the ductility and the dissipation capacity of the structures. Hence, between the ductility and the design forces...



The role of ductility

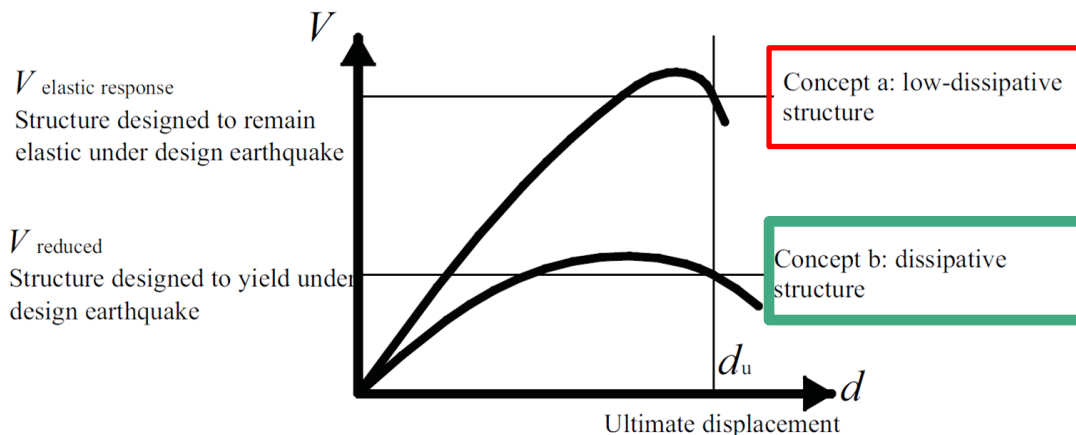
Most common strategy: increase hysteresis

Concept A: Structures made of sufficiently large sections that are subject to only elastic stresses (element failure is not ductile. In this case the structure's global behaviour is '**brittle**')

Concept B: Structures made of smaller sections, designed to form numerous **plastic zones** (hinges), intentionally designed to undergo cyclic plastic deformations without failure, and the structure as a whole is designed such that only those selected zones (hinges) will be plastically (nonlinear) deformed.

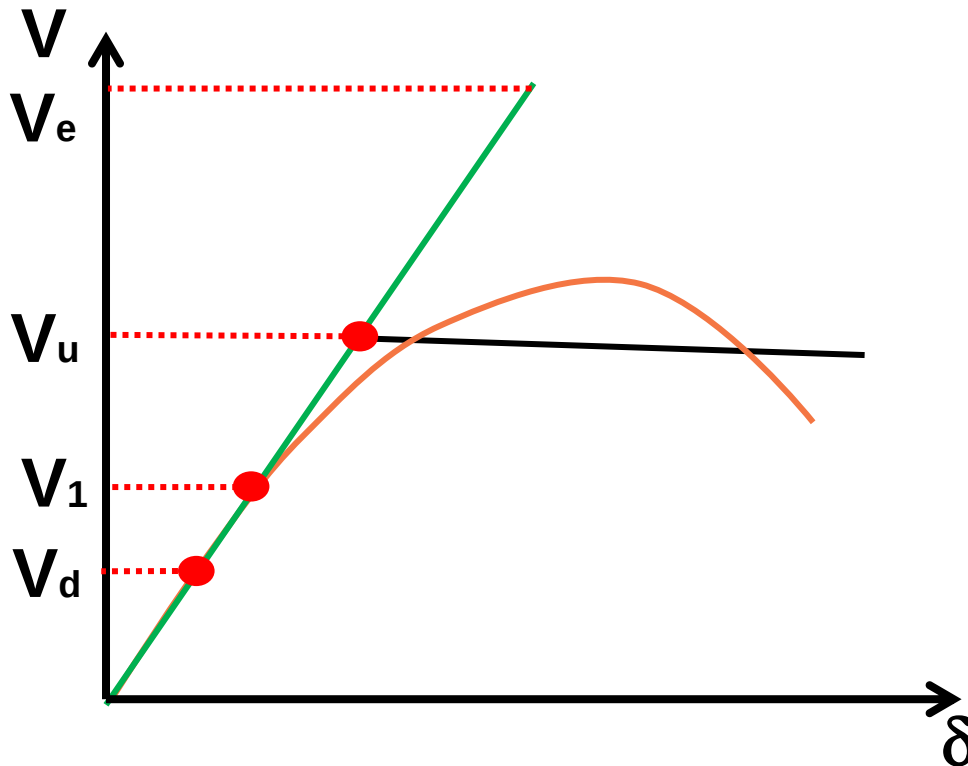
(EC8-1 Table 6.1)

Design concept	Structural ductility class	Range of the reference values of the behaviour factor q
Concept a) Low dissipative structural behaviour	DCL (Low)	$\leq 1,5 - 2$
Concept b) Dissipative structural behaviour	DCM (Medium)	≤ 4 also limited by the values of Table 6.2
	DCH (High)	only limited by the values of Table 6.2



The role of ductility

In EC8 the **equivalent of the ductility factor is called q (behaviour factor)**. This may be, more in general, expressed as the ratio between the force demand on an equivalent elastic structure over the force at first yielding of the inelastic structure



$$q = \frac{(S_{pa})_u}{(S_{pa})_y}$$

$$q = \frac{V_e}{V_d} = \frac{V_e}{V_u} \frac{V_u}{V_1} \frac{V_1}{V_d} = q_D q_R q_S$$

Ductility component

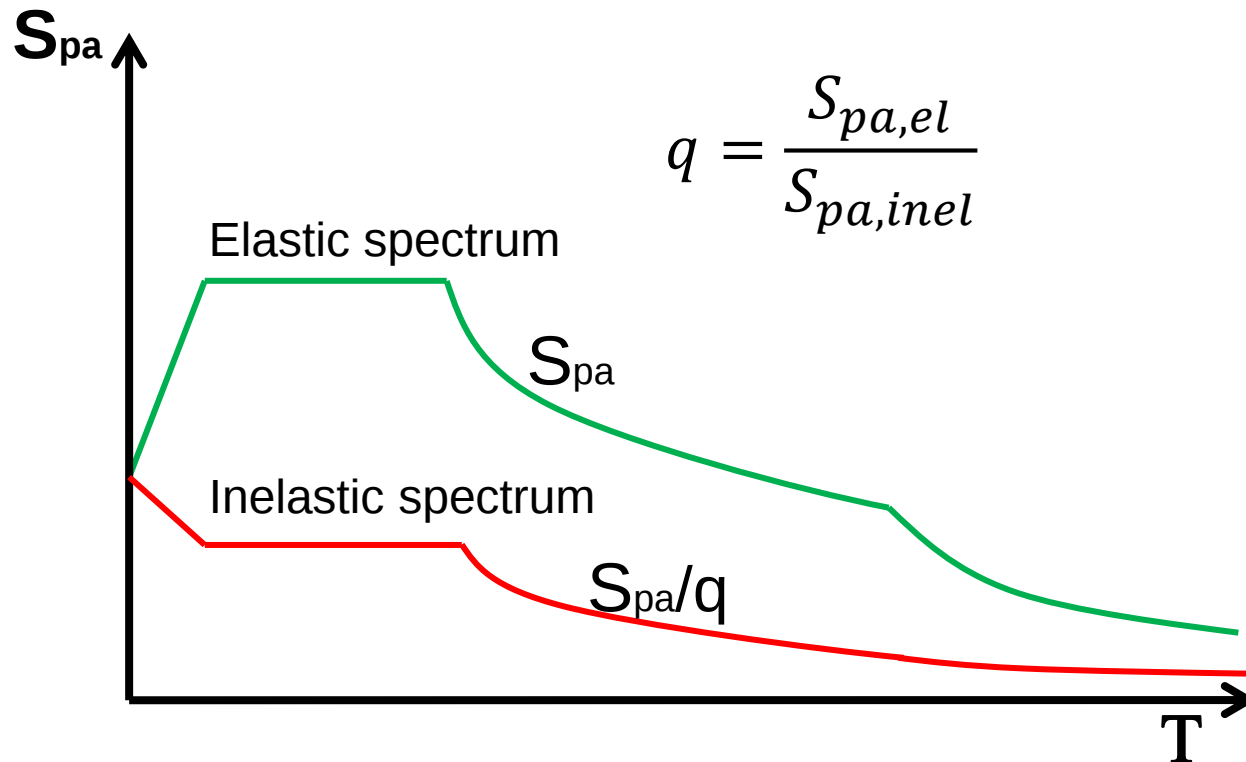
Redundancy & Redistribution

Design overstrength

The role of ductility

SDOFs: resisting force f_s – non linear system

The **behaviour factor scales the elastic spectrum** to obtain the design forces on the structure when this is assumed to work in non-linear range. Obviously I do not have to forget to design the structure to be **sufficiently ductile**. This depends on the **structural typology and detailing** rules. The structure has to be designed to be ductile both at **local and global level**.



The role of ductility

The **capacity of structural systems** to resist seismic actions in the non-linear range permits their design for resistance to seismic forces smaller than those corresponding to a linear elastic response.

The energy dissipation capacity of the structure is taken into account mainly through the ductile behaviour of its elements by performing a linear analysis based on a **reduced response spectrum**, called **design spectrum**.

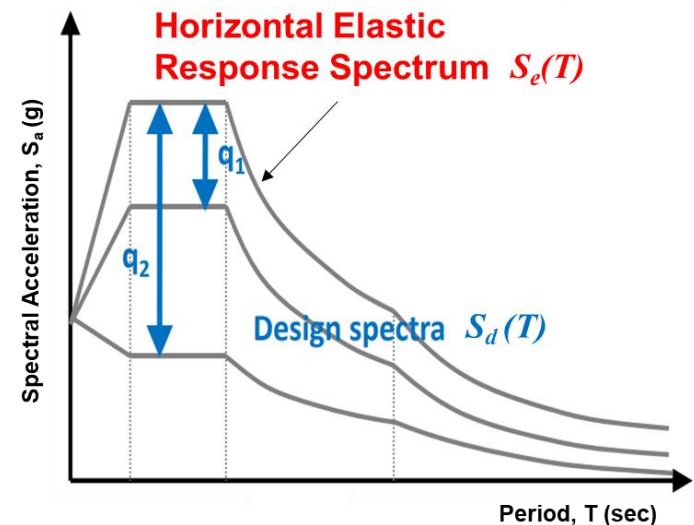
This reduction is accomplished by introducing the **Behaviour Factor** (q).

$$0 \leq T \leq T_B : S_d(T) = a_g \cdot S \cdot \left[\frac{2}{3} + \frac{T}{T_B} \cdot \left(\frac{2,5}{q} - \frac{2}{3} \right) \right]$$

$$T_B \leq T \leq T_C : S_d(T) = a_g \cdot S \cdot \frac{2,5}{q}$$

$$T_C \leq T \leq T_D : S_d(T) \begin{cases} = a_g \cdot S \cdot \frac{2,5}{q} \cdot \left[\frac{T_C}{T} \right] \\ \geq \beta \cdot a_g \end{cases}$$

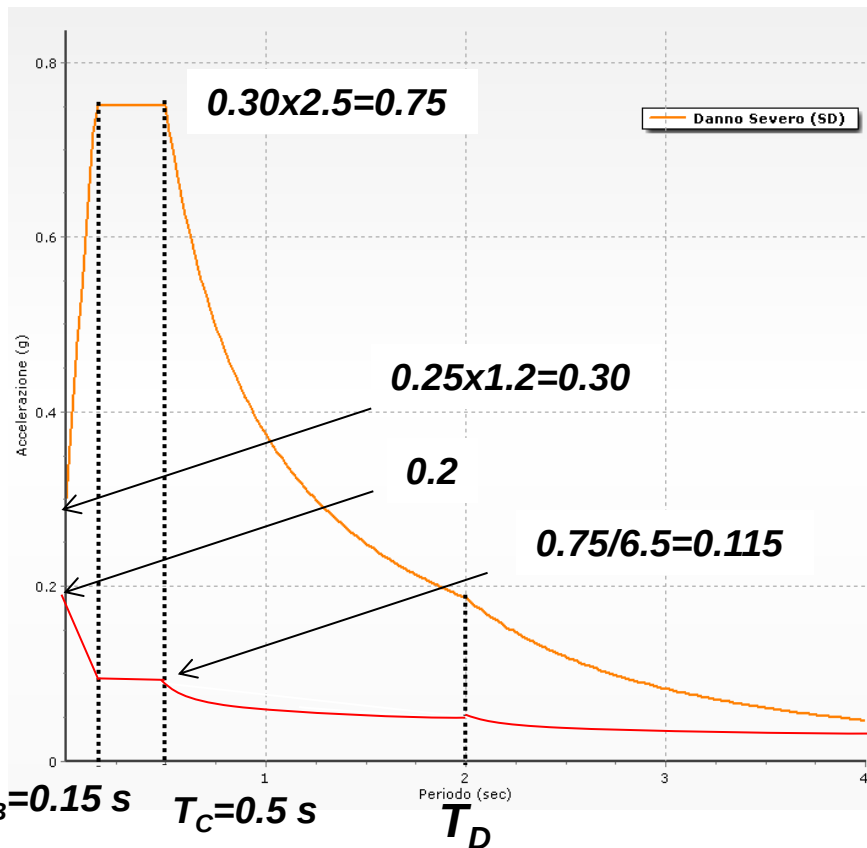
$$T_D \leq T : S_d(T) \begin{cases} = a_g \cdot S \cdot \frac{2,5}{q} \cdot \left[\frac{T_C T_D}{T^2} \right] \\ \geq \beta \cdot a_g \end{cases}$$



The role of ductility

EC8 inelastic design spectrum - example

$a_g=0.25g$ – medium hazard $S=1.20$ – soil type B (hard soil)



$$0 \leq T \leq T_B : S_d(T) = a_g \cdot S \cdot \left[\frac{2}{3} + \frac{T}{T_B} \cdot \left(\frac{2.5}{q} - \frac{2}{3} \right) \right]$$

$$T_B \leq T \leq T_C : S_d(T) = a_g \cdot S \cdot \frac{2.5}{q}$$

$$T_C \leq T \leq T_D : S_d(T) \begin{cases} = a_g \cdot S \cdot \frac{2.5}{q} \cdot \left[\frac{T_C}{T} \right] \\ \geq \beta \cdot a_g \end{cases}$$

$$T_D \leq T : S_d(T) \begin{cases} = a_g \cdot S \cdot \frac{2.5}{q} \cdot \left[\frac{T_C T_D}{T^2} \right] \\ \geq \beta \cdot a_g \end{cases}$$

The role of ductility

EC8 inelastic design spectrum - example

$a_g=0.25g$ – medium hazard $S=1.20$ – soil type B (hard soil)

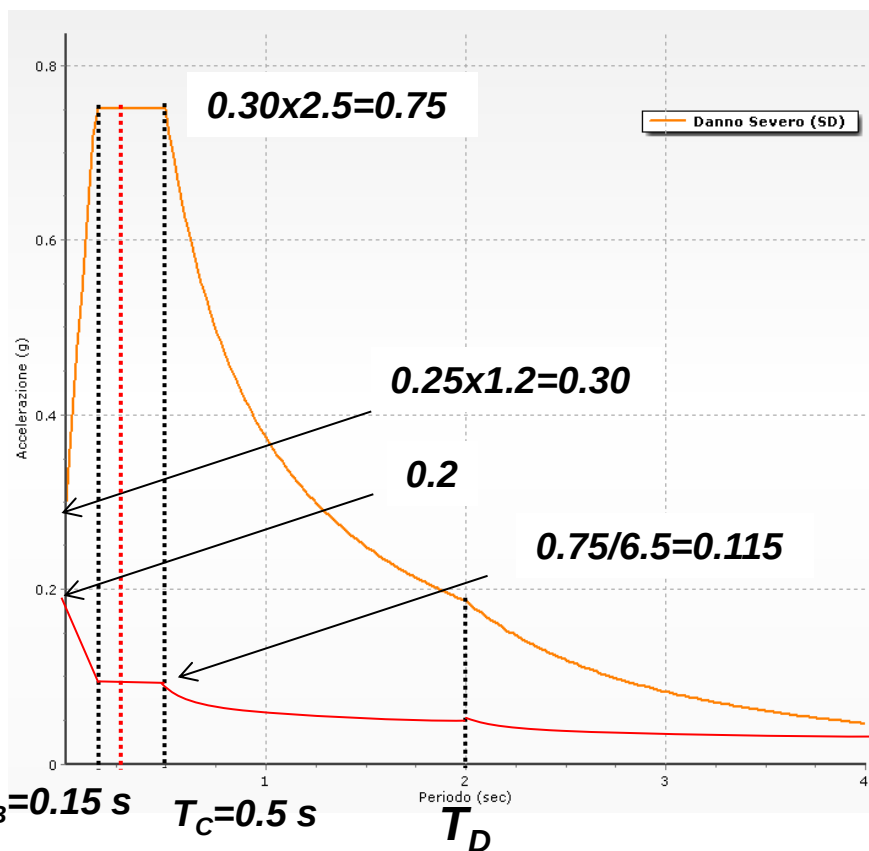
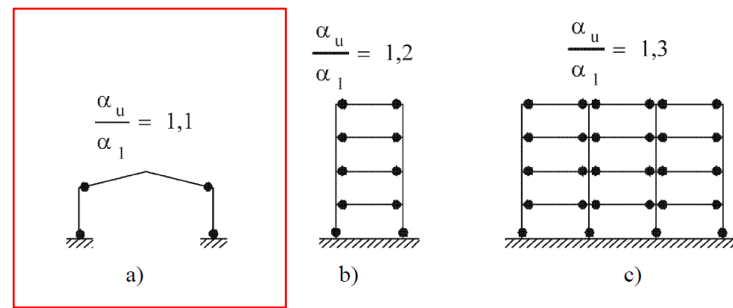


Table 6.2: Upper limit of reference values of behaviour factors for systems regular in elevation

STRUCTURAL TYPE	Ductility Class	
	DCM	DCII
a) Moment resisting frames	4	$5\alpha_u/\alpha_1$
b) Frame with concentric bracings		
Diagonal bracings	4	4
V-bracings	2	2,5
c) Frame with eccentric bracings	4	$5\alpha_u/\alpha_1$
d) Inverted pendulum	2	$2\alpha_u/\alpha_1$
e) Structures with concrete cores or concrete walls	See section 5	
f) Moment resisting frame with concentric bracing	4	$4\alpha_u/\alpha_1$
g) Moment resisting frames with infills		
	Unconnected concrete or masonry infills, in contact with the frame	2
	Connected reinforced concrete infills	See section 7
Infills isolated from moment frame (see moment frames)	4	$5\alpha_u/\alpha_1$

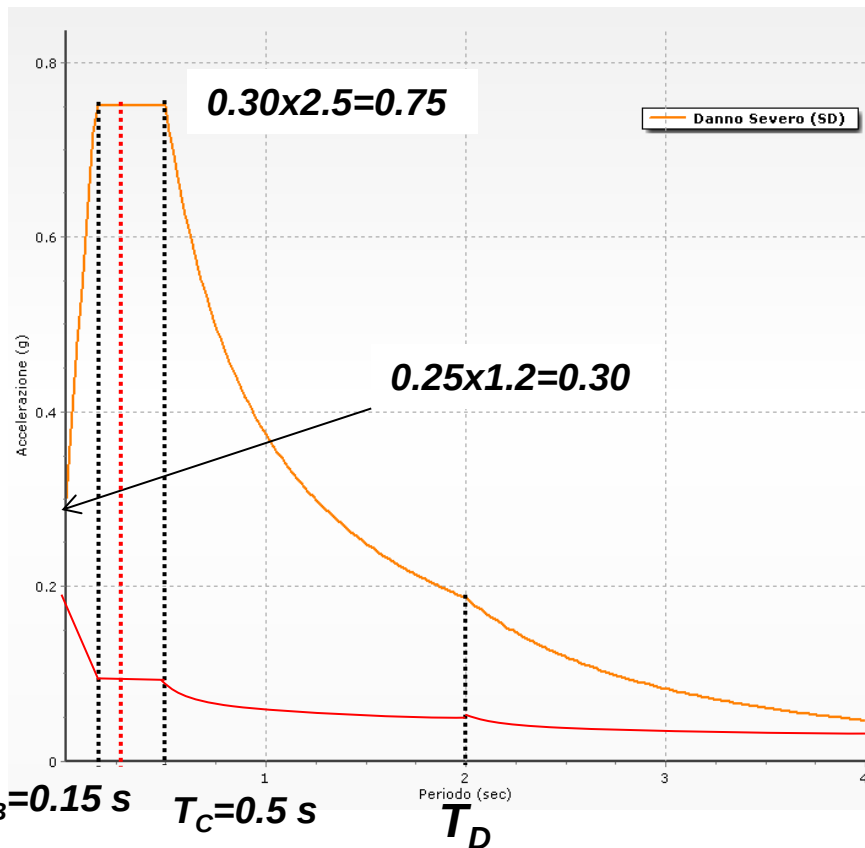


The role of ductility

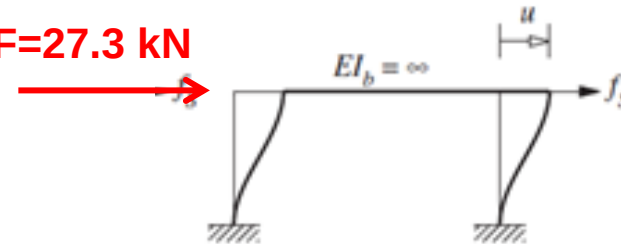
EC8 inelastic design spectrum - example

$a_g=0.25g$ – medium hazard $S=1.20$ – soil type B (hard soil)

$$V_{bo} = f_{So} = mA$$



$F=27.3\text{ kN}$



$$k = \sum_{\text{columns}} \frac{12EI_c}{h^3} = 24 \frac{EI_c}{h^3}$$

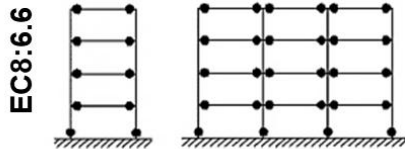
$$V_b = w \frac{S_e}{gq} = \frac{200.000 \times 0.75}{5.5} = 27270\text{ N}$$

The design forces are now **reduced by a factor equal to 5.5**. This means the structure will need to have a large **ductility** which needs to be properly designed (**hierarchical design and detailing rules to have large local/global deformation capacity**).

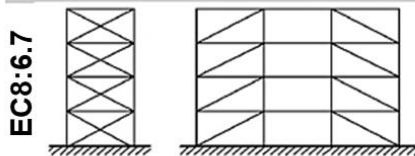
The role of ductility

Steel Structures

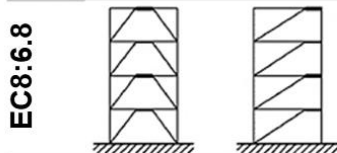
Moment Resisting Frames (MRF)



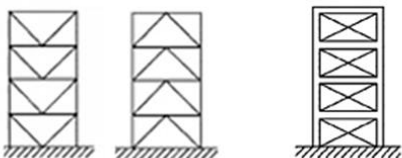
Diagonal braced frames (CBF)



Eccentric braced frames (EBF)



V Bracing (CBF) MRF + CBF

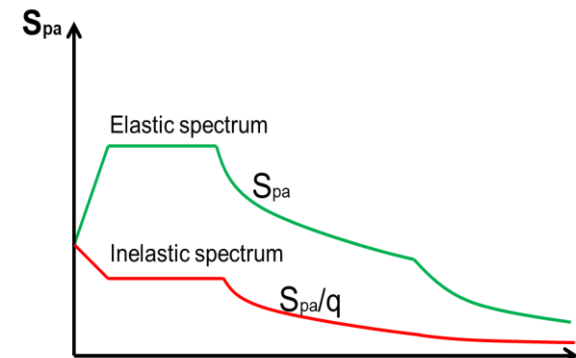
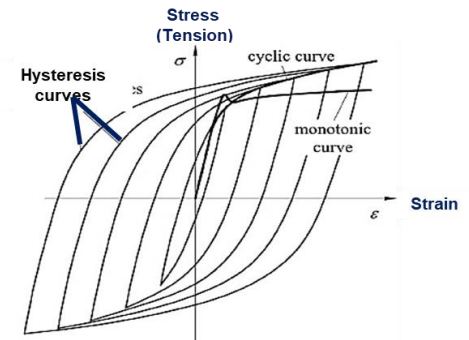


The behaviour factor q , introduced in **EC 8 3.2.2.5**, accounts for the energy dissipation capacity of the structure.

Upper limit of behaviour factor (q) for systems regular in elevation

STRUCTURAL TYPE	Ductility Class	
	DCM	DCH
a) Moment resisting frames MRF	4	$5\alpha_w/\alpha_1$
b) Frame with concentric bracings CBF	4	4
	2	2,5
c) Frame with eccentric bracings EBF	4	$5\alpha_w/\alpha_1$
d) Inverted pendulum	2	$2\alpha_w/\alpha_1$
e) Structures with concrete cores or concrete walls	See section 5	
f) Moment resisting frame with concentric bracing	4	$4\alpha_w/\alpha_1$
g) Moment resisting frames with infills		
	Unconnected concrete or masonry infills, in contact with the frame	
	2	2
Connected reinforced concrete infills	See section 7	
	4	$5\alpha_w/\alpha_1$
Infills isolated from moment frame (see moment frames)		

Values reduced by 20% if the building is irregular in elevation



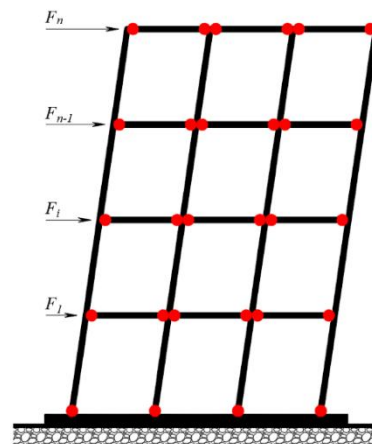
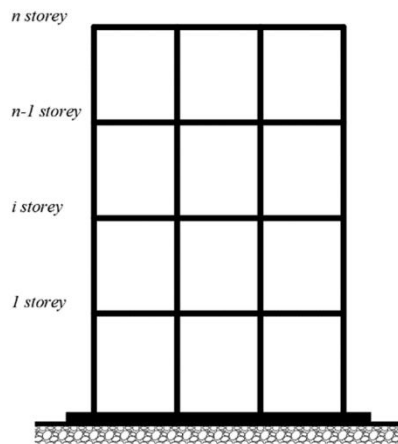
The role of ductility

RC Structures

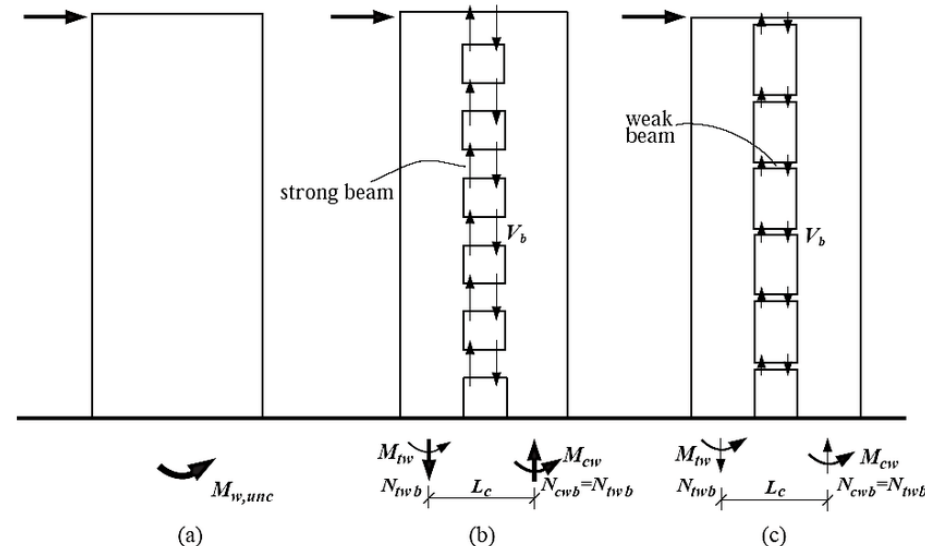
Table 5.1: Basic value of the behaviour factor, q_0 , for systems regular in elevation

STRUCTURAL TYPE	DCM	DCH
Frame system, dual system, coupled wall system	$3,0\alpha_w/\alpha_1$	$4,5\alpha_w/\alpha_1$
Uncoupled wall system	3,0	$4,0\alpha_w/\alpha_1$
Torsionally flexible system	2,0	3,0
Inverted pendulum system	1,5	2,0

$$q = q_0 k_w \geq 1,5$$



Frames (MRF)



Uncoupled wall

Coupled walls

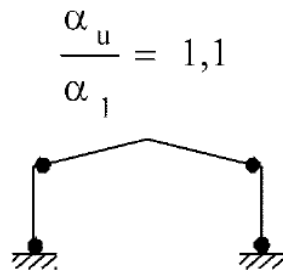
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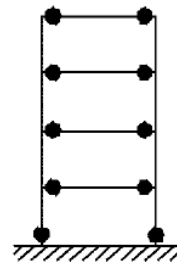
For frames



a)

*Single storey
Single bay*

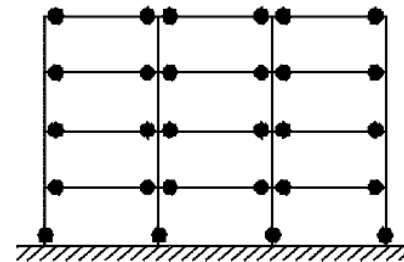
$$\frac{\alpha_u}{\alpha_1} = 1,2$$



b)

*Multi storey
Single bay*

$$\frac{\alpha_u}{\alpha_1} = 1,3$$



c)

*Multi storey
Multi bay*

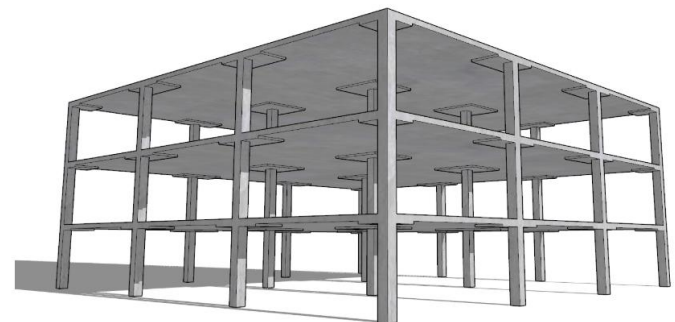
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- g) large walls structures: wall structures with at least two large walls in the horizontal direction of interest, which collectively support at least 20% of the total gravity load and have a fundamental fixed base period T_1 not greater than T_c . It is sufficient to have only one wall meeting these conditions in one of the two directions, provided that: (a) the basic value of the behaviour factor, q , in that direction is divided by a factor of 1,5 over the value in Table 10.1 and (b) there are at least two walls meeting these conditions in the orthogonal direction.
- h) flat slab structure: those composed of flat slabs considered in the design as primary seismic elements, supported directly by columns, in which the lateral loads are mostly resisted by the slab-column mechanism.

Large walls structures are included in the new generation of EC8



Flat slab structures are included in the new generation of EC8



The role of ductility

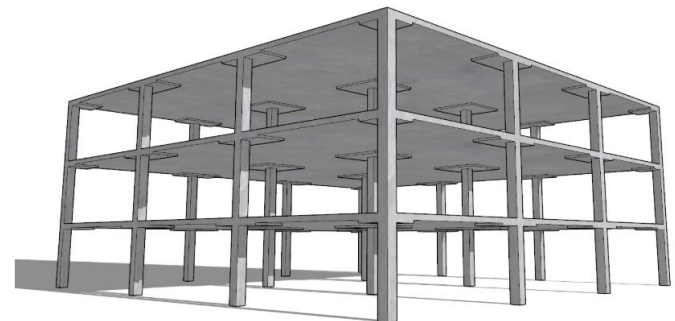
Structural type		q_R	q_D		$q = q_R q_S q_D$	
			DC2	DC3	DC2	DC3
Moment resisting frame or moment resisting frame-equivalent dual structures	multi-storey, multi-bay moment resisting frames or moment resisting frame-equivalent dual structures	1,3	1,3	2,0	2,5	3,9
	multi-storey, one-bay moment resisting frames	1,2			2,3	3,6
	one-storey moment resisting frames	1,1			2,1	3,3
Wall- or wall-equivalent dual structures	wall-equivalent dual structures	1,2	1,3	2,0	2,3	3,6
	coupled walls structures	1,2	1,4		2,5	3,6
	uncoupled walls structures	1,0	1,3		2,0	3,0
	large walls structures	--	--	--	$3,0 k_w$	
Flat slab structures		1,1	1,2	--	2,0	--

(2) To account for the prevailing failure mode in large walls structures, the behaviour factor q should be calculated taking into consideration the factor k_w given by Formula (10.1)

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$$k_w = \begin{cases} 0,5 & \text{if } \alpha_0 \leq 0,5 \\ (1 + \alpha_0)/3 & \text{if } 0,5 < \alpha_0 < 2,0 \\ 1,0 & \text{if } \alpha_0 \geq 2,0 \end{cases} \quad (10.1)$$

where:

α_0 is the prevailing aspect ratio (height-to-length ratio) of the walls of the structural system, which may be calculated by Formula (10.2):

$$\alpha_0 = \sum h_{wi} / \sum l_{wi} \quad (10.2)$$

where:

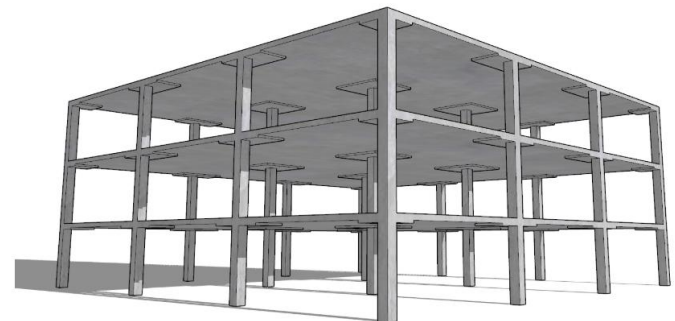
h_{wi} is the height of wall i ;

l_{wi} is the length of the section of wall i .

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Flat slab structures are included in the new generation of EC8



UZ PRIEKSU INŽENIERIJĀ

Paldies par uzmanību un sadarbību!

*Sirsnīgi pateicamies visiem Latvijas inženieriem par izrādīto
interesi un sniegto uzticēšanos.*

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